Loss of energy i.e. attenuation (concept of skin depth)

As the general wave equation in a charge free and external current free medium is given by

$$\nabla^2\vec{E} - \varepsilon\mu \frac{\partial^2\vec{E}}{\partial t^2} - \sigma\mu\frac{\partial\vec{E}}{\partial t} = 0 \quad ...(i) \qquad \text{and} \qquad \nabla^2\vec{H} - \varepsilon\mu \frac{\partial^2\vec{H}}{\partial t^2} - \sigma\mu\frac{\partial\vec{H}}{\partial t} = 0 \quad ...(ii)$$

to solve these equation, consider the solution be

$$\vec{E} = \vec{E}_0\, e^{i(\vec{k}.\vec{r} - \omega t)} \quad ...(iii) \qquad \text{and} \qquad \vec{H} = \vec{H}_0\, e^{i(\vec{k}.\vec{r} - \omega t)} \quad ...(iv)$$

so $$\frac{\partial\vec{E}}{\partial t} = -i\omega\,\vec{E}_0\, e^{i(\vec{k}.\vec{r} - \omega t)} = -i\omega E \quad ...(v)$$

and $$\frac{\partial^2\vec{E}}{\partial t^2} = (-i\omega)^2\, E_0\, e^{i(\vec{k}.\vec{r} - \omega t)} = \omega^2\vec{E} \quad ...(vi)$$

$$\nabla^2\vec{E} = \frac{\partial^2\vec{E}}{\partial r^2} = i^2\, k^2\,\vec{E} = -k^2\,\vec{E} \quad ....(vii)$$

substituting (v), (vi) and (vii) in (i)

$$-k^2\,\vec{E} + \varepsilon\mu\omega^2\,\vec{E} + i\sigma\omega\mu\,\vec{E} = 0$$

$$\left[k^2 - \varepsilon\mu\omega^2 - i\sigma\mu\varepsilon\right]\vec{E} = 0$$

or $$k^2 = \varepsilon\mu\omega^2 + i\sigma\mu\omega \qquad \text{or} \qquad k^2 = \varepsilon\mu\omega^2\left[1 + \frac{i\sigma}{\varepsilon\omega}\right] \quad ...(viii)$$

hence $\vec{k}$ is the propagation constant which represent the dispersion relation for any electromagnetic wave in a lossy dielectric medium and provides information about the nature of propagation of electromagnetic wave inside any medium.

$\vec{k}$ being a complex quantity so $k = A + iB$

$$k^2 = (A + iB)(A + iB) = A^2 - B^2 + i\,2AB \quad ...(ix)$$

Comparing equation (viii) and (ix)

$$A^2 - B^2 = \varepsilon\mu\omega^2 \quad \text{and} \quad 2AB = \sigma\mu\omega$$

On solving these two equations

$$A = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\left\{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{1/2}+1\right]^{1/2} \qquad \text{...(x)}$$

$$B = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\left\{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{1/2}-1\right]^{1/2} \qquad \text{...(xi)}$$

Here the term $\frac{\sigma}{\varepsilon\omega}$ is known as dissipation factor because it is the ratio of conduction current density to displacement current density. A is called attenuation constant and B is called as phase constant. In any loss less medium, waves do not attenuate so A = 0.

Hence equations for $\vec{E}$ and $\vec{H}$ can be written as

$$\vec{E} = \vec{E}_0\, e^{-Br}\, e^{i(Ar-\omega t)} \quad \text{...(xii)} \quad \text{and} \quad \vec{H} = \vec{H}_0\, e^{-Br}\, e^{i(Ar-\omega t)} \quad \text{...(xiii)}$$

here e^{-Br} is called attenuation factor and $e^{i(Ar-\omega t)}$ is called phase factor. e^{-Br} shows exponential decrease in amplitude with increase in r. Here B is called the absorption coefficient and is a measure of attenuation. For a good conducting medium $\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$ is negligible as displacement current does not exist experimentally. Hence general wave equation reduces to

$$\nabla^2\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} = 0 \qquad \text{...(i)}$$

and the general solution for above is $\vec{E} = \vec{E}_0\, e^{-Br}\, e^{i(Ar-\omega t)}$ ...(ii)

for a good conductor, when the frequency of the electromagnetic wave is not very high then $\sigma >> \varepsilon\omega$ i.e. $\frac{\sigma}{\varepsilon\omega} >> 1$.

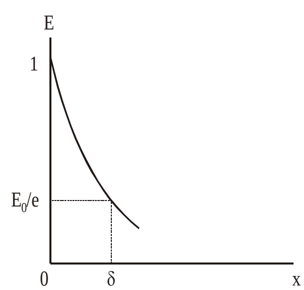
and $$A = B = \omega\sqrt{\frac{\mu\varepsilon}{2} \times \frac{\sigma}{\varepsilon\omega}} = \sqrt{\frac{\mu\sigma\varepsilon}{2}} = \frac{1}{\delta}\,(\text{assume})$$

so $$\delta = \sqrt{\frac{2}{\mu\sigma\varepsilon}} \qquad \text{...(iii)}$$

Here general solution becomes

$$\vec{E} = \vec{E}_0\, e^{-\frac{r}{\delta}}\, e^{i\left(\frac{r}{\delta} - \omega t\right)} \qquad \left[\text{as } A = B = \frac{1}{\delta}\right] \qquad \text{...(iv)}$$

Hence the amplitude of the electromagnetic wave becomes $\vec{E}_0 e^{-r/\delta}$. If $r = \delta$ then amplitude becomes $\vec{E}_0 e^{-1}$ or $\frac{\vec{E}_0}{e}$. Hence we can define distance $r = \delta$ as depth of penetration or skin depth, where the amplitude of electric field reduces to 1/e times of the amplitude of electric field at the surface (i.e. $r = 0$).



Attenuation of electric field in a conductor

Hence the skin depth is given as

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \qquad \text{...(v)}$$

The skin depth decrease with increase in frequency of the electromagnetic wave and the conductivity of the medium.

Physical significance of skin depth

It shows that any electromagnetic wave with high frequency (ω) cannot propagate through the conducting media. The large value of skin depth signifies the less attenuation of electromagnetic waves in any medium.