

MPHYCCJO-16

Anomalous Zeeman effect: \rightarrow The fine-structure component of multiplet spectral line, in presence of external magnetic field show complex Zeeman pattern. This is anomalous Zeeman effect. Zeeman splitting is smaller than the fine structure splitting.

Explanation of Normal Zeeman effect: \rightarrow

Normal Zeeman effect can be explained from classical electron theory as well as through quantum theory without noticing electron spin.

In terms of quantum theory, polyelectronic atom possesses an orbital angular momentum \vec{L} and orbital magnetic moment $\vec{\mu}_L$. Then

$$\text{Gyromagnetic ratio} = \frac{|\vec{\mu}_L|}{|\vec{L}|} = \frac{e}{2m} \quad \dots \quad \textcircled{1} \quad \begin{array}{l} e = \text{charge on electron} \\ m = \text{mass of electron} \end{array}$$

The vector $\vec{\mu}_L$ is directed opposite to the vector \vec{L} because the e^- is -vely charged.

In the presence of external magnetic field \vec{B} along the \vec{L} axis the vector \vec{L} precesses around the field direction having quantized component L_z is

$$L_z = M_L \frac{h}{2\pi} \quad \dots \quad \textcircled{2}$$

where $M_L = L, L-1, \dots, -L = 2L+1$ values.

This precession is known as Larmor precession. i.e. in presence of magnetic field L -level split into $2L+1$ levels

By Larmor theorem the angular velocity of precession

$$\omega = \frac{|\mu_L|}{|\vec{L}|} B = \frac{e}{2m} \cdot B \quad \dots \quad \textcircled{3}$$

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The wave number separation between consecutive component is equal to the separation between consecutive Zeeman levels

$$\Delta \nu = \nu' = \frac{eB}{4\pi mc}$$

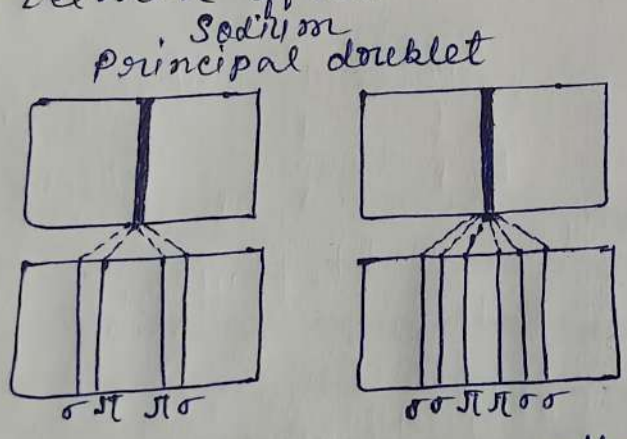
$e = 1.6 \times 10^{-19} \text{ C}$, $m = 9.1 \times 10^{-31} \text{ kg}$, $c = 3.0 \times 10^8 \text{ m s}^{-1}$

we get $\Delta \nu = 46.7 B \text{ m}^{-1}$ where B is in Tesla (N/A.m)

Normal Zeeman effect is fully explained.

Anomalous Zeeman effect: →

The fine structure component of a multiplet spectral line show a complex Zeeman pattern
 example: - D_1 & D_2 components of yellow doublet give four & six lines respectively in the Zeeman pattern. This is known as anomalous Zeeman effect.

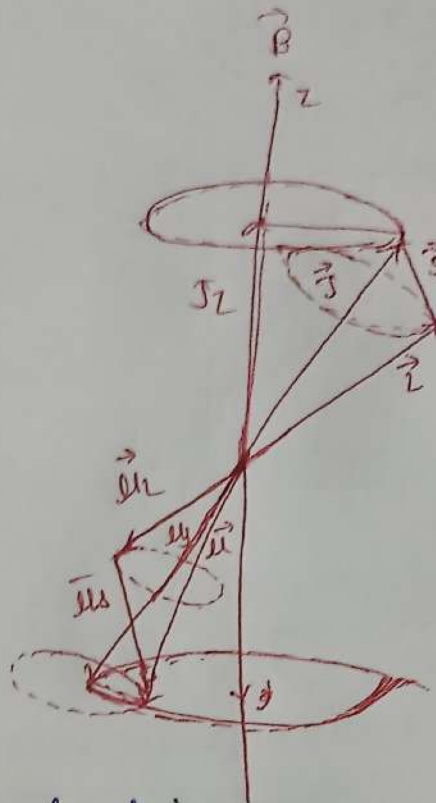


Anomalous Zeeman pattern

Explanation of Anomalous Zeeman effect: →

Anomalous Zeeman effect could be explained taking into account the spin of the electron.

In the vector atom model, the orbital angular momentum vector \vec{L} & the spin angular momentum vector \vec{S} precess rapidly around the total angular momentum vector \vec{J} as shown in the fig below



When the atom is placed in weak external magnetic field \vec{B} along the z-axis the magnetic moment of the atom associated with the total angular momentum causes the vector \vec{J} to precess slowly around the field called Larmor precession. component of \vec{J} along z axis i.e. J_z takes discrete values

$$J_z = \frac{M_j \hbar}{2\pi} \quad \text{① where } M_j = 0, J-1, J-2, \dots, -J = 2J+1 \text{ values}$$

The discrete orientation of atom in space and the small change in energy due to precession of \vec{J} around \vec{B} break each J level into $(2J+1)$ Zeeman levels.

$$\frac{|\vec{\mu}|}{|\vec{L}|} = \frac{e}{2m} \quad \text{--- ②} \quad \ll \quad \frac{|\vec{\mu}_s|}{|\vec{S}|} = \frac{2e}{2m} \quad \text{--- ③}$$

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It is clear that $\frac{|\vec{\mu}_L|}{|\vec{L}|} \neq \frac{|\vec{\mu}_S|}{|\vec{S}|}$ hence total magnetic moment $\vec{\mu} = \vec{\mu}_L + \vec{\mu}_S$ is not exactly antiparallel to \vec{J} .

\vec{J} is invariant hence $\vec{L}, \vec{S}, \vec{\mu}_L, \vec{\mu}_S$ & $\vec{\mu}$ precess around \vec{J}

$\mu_j =$ component of $\vec{\mu}_L$ along \vec{J} + component of $\vec{\mu}_S$ along \vec{J}

$$\mu_j = |\vec{\mu}_L| \cos(\angle \vec{L}, \vec{J}) + |\vec{\mu}_S| \cos(\angle \vec{S}, \vec{J})$$

$$\mu_j = \frac{e}{2m} |\vec{L}| \cos(\angle \vec{L}, \vec{J}) + \frac{ge}{2m} |\vec{S}| \cos(\angle \vec{S}, \vec{J}) \quad \dots (3)$$

$$|\vec{L}| = \sqrt{L(L+1)} \frac{h}{2\pi} \quad (4) \quad \& \quad |\vec{S}| = \sqrt{S(S+1)} \frac{h}{2\pi} \quad (5)$$

$$\cos(\angle \vec{L}, \vec{J}) = \frac{J(J+1) + L(L+1) - S(S+1)}{2\sqrt{J(J+1)}\sqrt{L(L+1)}} \quad \dots (6)$$

$$\cos(\angle \vec{S}, \vec{J}) = \frac{J(J+1) + S(S+1) - L(L+1)}{2\sqrt{J(J+1)}\sqrt{S(S+1)}} \quad \dots (7)$$

Putting (4), (5), (6), (7) into (3)

$$\mu_j = \frac{e}{2m} \left[\frac{J(J+1) + L(L+1) - S(S+1)}{2\sqrt{J(J+1)}} + \frac{J(J+1) + S(S+1) - L(L+1)}{\sqrt{J(J+1)}} \right] \frac{h}{2\pi}$$

$$\mu_j = \frac{e}{2m} \left[1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \right] \sqrt{J(J+1)} \frac{h}{2\pi} \quad \dots (8)$$

Lande g factor 'g' = $1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad \dots (9)$

using (9) in (8)

$$\mu_j = \frac{e}{2m} \cdot g \sqrt{J(J+1)} \frac{h}{2\pi} = g \frac{e}{2m} |\vec{J}| \Rightarrow \frac{\mu_j}{|\vec{J}|} = g \frac{e}{2m} \quad \dots (10)$$

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By Larmor's theorem

$$\omega = \frac{\mu_j}{(J)} B = g \frac{e}{2m} \cdot B$$

$$\Delta E = \omega J_z = g \frac{e}{2m} \cdot B M_j \cdot \frac{h}{2\pi} = g M_j \frac{eh}{4\pi m} \cdot B \quad \text{--- (11)}$$

In terms of wave number the interaction energy

$$-\Delta T = \frac{\Delta E}{hc} = g M_j \frac{eB}{4\pi mc} \quad , \quad L' = \frac{eB}{4\pi mc} = \text{Lorentz unit.}$$

$$\boxed{-\Delta T = g M_j L'} \quad \text{--- (12)}$$

J level split into $2J+1$ equispaced Zeeman levels corresponding to the possible values of M_j . The Zeeman splitting is different for different J levels & depends on g value for that level.