medium.

Skin Depth (δ) :-

If we incident EM wave on conductor. Inside the conductor, near the surface E-field is not zero but It will travell some distance & then E-field become zero.

Skin depth is the distance at which amplitud of EM wave become 1/e value of the value at the surface.

If E-field at surface is E_0 then after travelling some distance i.e. Skin depth, it become $\frac{E_0}{e}$.

More is β, less is the skin-depth.

$$\boxed{\delta = \frac{1}{\beta}}$$

Skin-depth of free space :- is ∞.

In free-space there is no decay in amp. of wave.

For Good Conductor,

$$\boxed{\delta_{good} = \frac{1}{\beta} = \sqrt{\frac{2}{\sigma \omega \mu}}}$$

for Bad Conductor,

(means it is Insulator)

$\boxed{\delta_{bad} = \infty}$ for perfect Insulator $\left\{\begin{matrix}\sigma = 0 \\ \beta = 0\end{matrix}\right\}$

for perfect dielectric (it is a " ")

If conductivity is finite but very small then there will be some skin depth. There will be decay so δ will be finite.

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[1+\frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2-1\right]^{1/2}$$

$$= \omega\sqrt{\frac{\varepsilon\mu}{2}}\ \frac{1}{\sqrt{2}}\left(\frac{\sigma}{\varepsilon\omega}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\boxed{\delta_{bad} = \frac{1}{\beta_{bad}} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$

{for perfect insulator, $\sigma = 0$ so $\delta = \infty$}

<u>Phase</u>:- If wave impedence is complex, it defines the phase difference b/w $\vec{E}$ & $\vec{B}$.

$$Z = \left|\frac{E}{H}\right| = \frac{\mu E}{B} = \frac{\mu\omega}{k^*} \qquad \left\{\frac{E}{B} = v = \frac{\omega}{k^*}\right.$$

$$= \frac{\mu\omega}{(\alpha+i\beta)} = \text{complex quantity}$$

Inside the conducting medium $\vec{E}$ & $\vec{B}$ are out of phase.

We know $K^* = \alpha + i\beta = Ke^{i\phi}$

where $K \rightarrow$ Amp. $\Rightarrow K = (\alpha^2+\beta^2)^{1/2}$

$\phi \rightarrow$ phase difference $\Rightarrow \tan\phi = \frac{\beta}{\alpha}$

We know that

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2}$$

So $K = (\alpha^2+\beta^2)^{1/2} = [\omega$

$$\boxed{K = \omega\sqrt{\mu\varepsilon}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

and $\tan\phi = \frac{\beta}{\alpha}$

$$\boxed{\phi = \tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

This is the phase diff. b/w $\vec{E}$ & $\vec{B}$. If $\sigma = 0$ then $\phi = 0$ i.e. No phase diff.

We have $\vec{B} = \frac{\vec{K}^* \times \vec{E}}{\omega}$

$\left\{ \begin{array}{l} \vec{K}^* = K^* \hat{z} \\ \vec{E} = E\hat{x} \\ \text{so } \vec{B} \text{ will be in } \hat{y} \end{array} \right.$

$$\vec{B} = \frac{K^* E}{\omega} \hat{y}$$

$$\vec{B} = \frac{K e^{i\phi}}{\omega} E_0 e^{-\beta(\hat{n}\cdot\hat{r})} e^{i(kz-\omega t)} \hat{y}$$

$\left\{ \begin{array}{l} K^* = K e^{i\phi} \\ E = E_0 e^{i(kz-\omega t)} \end{array} \right.$

On putting the values of K, We get

$$\boxed{\vec{B} = \sqrt{\mu\varepsilon} \left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E_0 e^{-\beta(\hat{n}\cdot\hat{r})} e^{i[kz-(\omega t-\phi)]} \hat{y}}$$

Phase of $\vec{E} = (kz - \omega t)$

$\vec{B} = [kz - (\omega t - \phi)]$

→ $\vec{B}$ is lagging behind $\vec{E}$ by phase diff. ϕ i.e. $\vec{E}$ is leading by ϕ.

→ The amplitude of $\vec{B}$ is greater that $\vec{E}$ (as $\frac{\sigma}{\omega\varepsilon} >> 1$) becoz, Amp. of $\vec{B}$ contains amp. of $\vec{E}$ & also a quantity multiplied by amp. of E (~~which~~ the quantity is greater than 1)

→ E decays faster that's why its amp. is small.

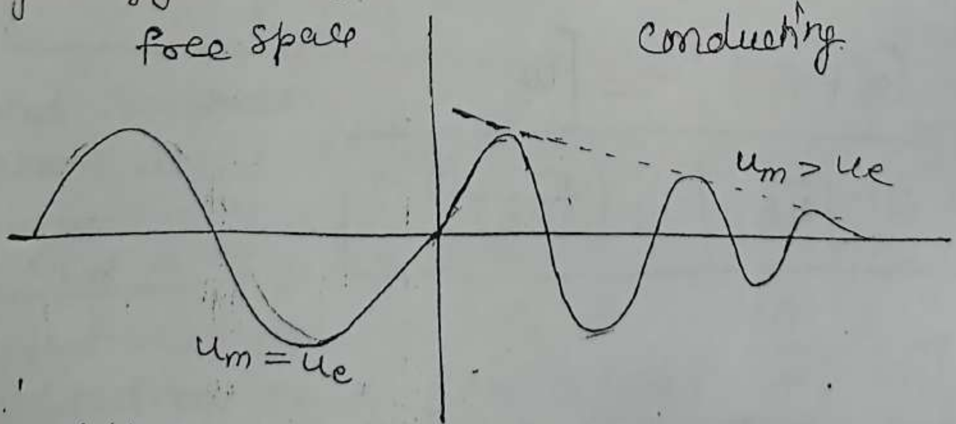
→ Energy density –

$$u_m > u_e$$

At any particular distance ↑

$$u_m \propto B^2 \ \& \ u_e \propto E^2$$

i.e. mag. energy density is greater then electric energy density.



→ Poynting Vector :- $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu} \hat{z}$

$\because \vec{E} \to \hat{x}, \vec{B} \to \hat{y}$

$\therefore S \to \hat{z}$

i.e. dirⁿ of energy flow is same as dirⁿ of wave propogation.

We have $\vec{B} = \frac{\vec{K}}{\omega}\vec{E}$.

$$\frac{\vec{B}}{E} = \sqrt{\mu\varepsilon}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}$$

so $\vec{S} = \sqrt{\frac{\varepsilon}{\mu}}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E^2\hat{z}$.

$$\vec{S} = \sqrt{\frac{\varepsilon}{\mu}}\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4} E_0^2 e^{-2\beta z}\left[e^{2i(Kz-\omega t)}\right]\hat{z}$$

<u>Conclusions in conducting medium</u>

① → Ele. & mag. field amplitude decays exponentially

$E_0 e^{-\beta z}$

$= \boxed{E_0 e^{-\frac{z}{\delta}}}$ $\quad (\delta = \frac{1}{\beta})$

$\vec{K} = K.\hat{n}$

$\hat{n}$ = dirⁿ of prop.

$(\hat{r}.\hat{n} = \hat{z})$

e.g. – given that the skin depth for a certain material $\delta = 10\ nm$. Calculate the amp. of $\vec{E}$ after 100nm distance into the conductor.

E_0 – field at surface.

Z → distance travelled into the medium.

$$E_0 e^{-\frac{z}{\delta}} = E_0 e^{-\frac{100\ nm}{10\ nm}} = E_0 e^{-10}$$

e^{-10} → very very small (negligible)

* If skin depth is given & find the amp. after a distance the use above formula.

② → $E \perp K$, $B \perp K$

i.e. $\boxed{E \perp B \perp K}$

EM waves are transvers.

③ → There is a phase diff. b/w $\vec{E}$ & $\vec{B}$ fields.

$\vec{E}$ leading in phase by angle $\phi = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$

④ → $u_m \neq u_e$ but $u_m > u_e$

so most of the energy lies in the mag. field

& decay of u is $\boxed{u \propto e^{-\frac{2z}{\delta}}}$