

Dr. Ranjana Singh  
 Assistant Professor, Physics  
 Haz Prasad Das Jain College  
 Ara, Bihar, India

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MPHYCC10-15

Nuclear spin & hyperfine splitting: →

Isotope effect failed to explain the hyperfine splitting in many cases. It was seen that the number of hyperfine components is often greater than the number of isotopes. It was also observed that certain element having single isotope, still have hyperfine structure. Pauli in 1924 pointed out that hyperfine structure can be explained by assuming the atomic nucleus contains an intrinsic spin angular momentum  $I$ . Magnetic dipole moment  $\vec{\mu}_I$  is associated with  $\vec{I}$ . magnitude of nuclear angular momentum is

$$|\vec{I}| = \sqrt{I(I+1)} \frac{h}{2\pi} \quad (1) \quad I = \text{nuclear spin quantum number.}$$

Angular momentum has different magnitude for different nuclei. Also it is different ~~from~~ for different isotopes of the same element.

Just as  $\vec{L}$ ,  $\vec{S}$  &  $\vec{J}$  have quantized component along an axis in space, the component of  $\vec{I}$  along the z axis is

$$I_z = M_I \frac{h}{2\pi} \quad (2)$$

possible values of magnetic quantum number  $M_I$  is  $2I+1$  where  $M_I = I, I-1, I-2, \dots, -I$

Nuclear proton/neutron motions produces magnetic moment  $\vec{\mu}_I$ .

$$\vec{\mu}_I = g_I \left( \frac{e}{2m_p} \right) \vec{I} \quad (3) \quad \begin{matrix} e = \text{charge on proton} \\ m_p = \text{mass of proton} \end{matrix} \quad g_I = \text{nuclear g factor.}$$

$$\mu_I = g_I \left( \frac{e}{2m_p} \right) \sqrt{I(I+1)} \frac{h}{2\pi} \quad (4)$$

$$\mu_I = g_I \frac{e h}{4\pi m_p} \sqrt{I(I+1)} = g_I \sqrt{I(I+1)} \mu_N \quad (5)$$

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$$\mu_N = \frac{eh}{4\pi m_p} = \textcircled{6} \text{ nuclear magneton}$$

proton mass = 1836 times mass of electron

hence  $\mu_N = \frac{1}{1836} \times \mu_e$  -  $\textcircled{7}$   $\mu_e =$  Bohr magneton.

component of  ~~$\mu$~~   $\mu_I$  along z axis

$$\mu_{Iz} = g_I M_I \mu_N - \textcircled{8}$$

maximum value of  $M$  is  $I$

Improved atomic vector model :  $\rightarrow$

Now ~~to~~ the atomic vector model constructed taking the nuclear spin into account. The total angular momentum of the atom is the vector sum of three angular momenta

- (a) electron orbital angular momentum  $\vec{L}$
- (b) electron spin angular momentum  $\vec{S}$
- (c) Nuclear spin angular momentum  $\vec{I}$

Total angular momentum  $\vec{F}$  is

$$\vec{F} = \vec{L} + \vec{S} + \vec{I} = \vec{J} + \vec{I}$$

Due to interaction between electron orbit and spin,  $\vec{L}$  &  $\vec{S}$  precess rapidly around resultant  $\vec{J}$ .

Further, the interaction between the nuclear magnetic moment and magnetic field produced by the orbital and spin motions, the atomic electrons couples  $\vec{I}$  with  $\vec{J}$  causes these vectors to precess around their resultant  $\vec{F}$ .

Precession of  $\vec{I}$  &  $\vec{J}$  is 1000 times smaller than  $\vec{L}$  &  $\vec{S}$  about  $\vec{J}$  hence the energy difference are very much smaller.



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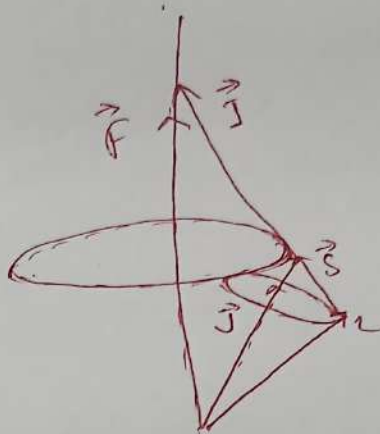
The quantized values of the angular momentum  $\vec{F}$  are

$$\sqrt{F(F+1)} \frac{h}{2\pi} \quad F = \text{hyperfine splitting quantum number}$$

$$F = J+I, J+I-1, \dots, |J-I|$$

$$I \geq J \text{ then } F = 2J+1$$

$$I < J \text{ then } F = 2I+1$$



Interaction energy:  $\rightarrow$  The interaction energy due to  $\vec{I}$  &  $\vec{J}$  can be given as

$$E_{I,J} = \frac{1}{2} A' [F(F+1) - I(I+1) - J(J+1)] \quad \text{--- (1)}$$

$A' = \text{constant}$

The various hyperfine split level of a given term value for the given atom have same  $I$  &  $J$  value but different  $F$  value.

Hence the separation between two hyperfine split levels can be obtained by substituting first  $F+1$  then  $F$ . The energy separation between  $F+1$  &  $F$  values

$$\boxed{\Delta E' = A'(F+1)} \quad \text{--- (2)}$$

The order of hfs levels in some hypermultiplets is normal i.e. smallest  $F$  level deepest while in some it is inverted i.e. largest  $F$  level is deepest.

selection rule for  $f$ ,  $\Delta F = 0, \pm 1$  but  $F=0 \leftrightarrow F=0$

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Zeeman effect : It was invented by Zeeman in 1896. He observed that light source giving line spectrum placed in external magnetic field, the spectral lines emitted by the atomic source split into a number of polarised components. This effect of magnetic field on the atomic spectral lines is called "Zeeman effect".

There are two types of 'Zeeman effect' observed in the presence of external magnetic field.

Normal Zeeman effect :- In the presence of external magnetic field a singlet spectral line viewed at right angles to the external magnetic field direction split into three plane-polarised components.

A central unshifted line with electric vector vibrating parallel to the magnetic field is known as  $\pi$  component.  
Two other lines equally displaced on either side with electric vector vibrating perpendicular to the magnetic field is known as  $\sigma$  component.

