Distinction b/w Good Conductor & Bad Conductor :-

Conductor is good or bad - It depands upon its

→ Conductivity (σ)

→ Relaxation time (τ)

if we put a free static charge inside the conductor then how quickely that charge come out to the surface - that time will be the Relaxation time.

If conductor is bad, Its relaxation time will be more, it take more time to come out to the surface.

Relaxation time is given by permitivity to conductivity ratio.

$$\tau = \frac{\varepsilon}{\sigma}$$

Now If we fall an EM wave over a conductor of conductivity σ & per. ϵ, we know $\vec{E}$ inside the conductor is 0. so amplitud of EM wave inside the conductor must be zero. This E. field is not electrostatic, it is dynamic (changing with time). Here ω is the freq. of oscillation of $\vec{E}$lec. & mag. field.

So Conductor is good or bad → it is Not only depand of σ & τ but also depand on freq. ω.

→ If a conductor is good for a perticular freq. ω, It may be bad for some another freq.

If $\tau << \frac{1}{\omega}$

$\Rightarrow$ Good Conductor

If $\tau >> \frac{1}{\omega}$ $\Rightarrow$ Bad Conductor

(i) $\Rightarrow \frac{\varepsilon}{\sigma} << \frac{1}{\omega} \Rightarrow \frac{\sigma}{\varepsilon\omega} >> 1$ $\Rightarrow$ good conductor

(ii) $\Rightarrow \frac{\sigma}{\varepsilon\omega} << 1$ $\Rightarrow$ bad conductor

Inside the conductor, free volume charge density

$\rho_f = 0$

At $t = 0$, we put ρ_f inside conductor & after some time ρ_f becomes 0. So have to find that time in which ρ_f become 0 from ρ_f.

Use Continuity eqn,

$$\vec{\nabla} \cdot \vec{J}_f = -\frac{\partial \rho_f}{\partial t}$$

We have, $J_f = \sigma E$

so $\sigma(\vec{\nabla} \cdot \vec{E}) = -\frac{\partial \rho_f}{\partial t}$

$$\Rightarrow \frac{\sigma}{\varepsilon} \rho_f = -\frac{\partial \rho_f}{\partial t}$$

$\nabla \cdot E = \frac{\rho_f}{\varepsilon}$

$\{$ $\nabla \cdot E = 0$ only when all the charge come out of surface

$$\Rightarrow \frac{\partial \rho_f}{\rho_f} = -\frac{\sigma}{\varepsilon} dt$$

$$\Rightarrow \ln \rho_f(t) = -\frac{\sigma}{\varepsilon} t + C$$

At time $t = 0$, $C = \ln \rho_f(0)$

$$\Rightarrow \ln \rho_f(t) = -\frac{\sigma}{\varepsilon} t + \ln \rho_f(0)$$

$$\Rightarrow \ln \frac{\rho_f(t)}{\rho_f(0)} = -\frac{\sigma}{\varepsilon} t$$

$$\Rightarrow \rho_f(t)/\rho_f(0) = e^{-\frac{\sigma}{\varepsilon} t}$$

$$\Rightarrow \boxed{\rho_f(t) = \rho_f(0) e^{-\frac{\sigma}{\varepsilon} t}}$$

It defines - How the free charge decays with time exponentially inside the conductor.

If We have a Perfect Conductor,

$\sigma = \infty$

So free charge inside $= 0$

Relaxation time $\tau = \frac{\varepsilon}{\sigma}$ (if $\sigma = \infty$)

$\boxed{\tau = 0}$

i.e. No time lag in putting the charge & come out to the surface.

for Insulator

$\sigma = 0$

$$\boxed{\tau = \frac{\varepsilon}{\sigma} = \infty}$$

i.e. charge take ∞ time to come out to the surface.

i.e. It can never come out to the surface.

Note :- $\vec{\nabla}\cdot\vec{D} = \rho_f$ is valid for every medium.

$\vec{\nabla}\cdot(\varepsilon\vec{E}) = \rho_f$ " " only for isotropic medium.

for semiconductor

Conductivity is small but finite.

σ = very small

so τ = large

i.e. It will take more time to come out.

- $\vec{\nabla}\cdot\vec{E} = 0$ is valid only for perfect conductor.

Wave velocity inside good & bad conductor :-

$$v_{good} = \frac{\omega}{\alpha}$$

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]^{1/2}$$

for good, $\frac{\varepsilon}{\sigma} << \frac{1}{\omega} \Rightarrow \frac{\sigma}{\varepsilon\omega} >> 1$ so neglect 1 as compare to $\frac{\sigma}{\varepsilon\omega}$

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]^{1/2}$$

$$\alpha_{good} = \beta_{good} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left(\frac{\sigma}{\omega\varepsilon}\right)^{1/2}$$

$$= \sqrt{\omega}\sqrt{\frac{\varepsilon\mu}{2}}\left(\frac{\sigma}{\varepsilon\omega}\right)^{1/2} = \sqrt{\frac{\sigma\omega\mu}{2}}$$

$$\boxed{v_{good} = \frac{\omega}{\alpha} = \sqrt{\frac{2\omega}{\sigma\mu}}}$$

for bad cond. $\frac{\sigma}{\varepsilon\omega} << 1$

$$v_{bad} = \frac{\omega}{\alpha}$$

$$\alpha_{bad} = \omega\sqrt{\varepsilon\mu}$$

$$v_{bad} = \frac{\omega}{\omega\sqrt{\varepsilon\mu}}$$

$$\boxed{v_{bad} = \frac{1}{\sqrt{\varepsilon\mu}}}$$

This is similar to velocity inside the dielectric medium.