$$\Rightarrow \boxed{\nabla^2 E - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \quad \text{---} (5)$$

Take curl of eqⁿ (4),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu\sigma (\vec{\nabla} \times \vec{E}) + \mu\varepsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = \mu\sigma \left(-\frac{\partial \vec{B}}{\partial t}\right) + \mu\varepsilon \left(-\frac{\partial^2 \vec{B}}{\partial t^2}\right)$$

$$\boxed{\nabla^2 B - \mu\sigma \frac{\partial \vec{B}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0} \quad \text{---} (6)$$

eqⁿ (5) & (6) are IInd order differential eqⁿs.

IInd term, of eqⁿ (5) & (6) are damping terms. coms from damped harmonic oscillator. $\frac{d^2 x}{dt^2} - \omega^2 x = \underline{\frac{dx}{dt}}$

Bcoz of this term, amplitude of damped harmonic oscillator is decaying.

Now, we take wave vector <u>K is a complex quantity</u> in the solutions of wave eqⁿ.

$$\vec{E} = E_0 e^{i(\vec{K}^* \cdot \vec{r} - \omega t)}$$

$$\vec{B} = B_0 e^{i(\vec{K}^* \cdot \vec{r} - \omega t)}$$

These solutions must satisfy the given diff. eqⁿ.

$$(5) \Rightarrow (i K^*)^2 \vec{E} - (-i\omega)\mu\sigma \vec{E} - (-i\omega)^2 \mu\varepsilon \vec{E} = 0$$

$$-(K^*)^2 \vec{E} + i\omega\mu\sigma \vec{E} + \mu\varepsilon\omega^2 \vec{E} = 0$$

(from (6) → We get same eqⁿ)

$$\boxed{(K^*)^2 = \mu\varepsilon\omega^2 + i\omega\mu\sigma}$$

$$\boxed{K^* = [\mu\varepsilon\omega^2 + i\omega\mu\sigma]^{1/2}} \quad \text{———} (7)$$

Wave vector is Complex.

Let $K^* = \alpha + i\beta$

$\alpha \rightarrow$ real part
$\beta \rightarrow$ imaginary part

$$\boxed{(K^*)^2 = \alpha^2 - \beta^2 + 2i\alpha\beta} \quad \text{———} (8)$$

Compare (7) & (8) $\Rightarrow$

$$\alpha^2 - \beta^2 = \mu\varepsilon\omega^2 \quad \text{———} (9)$$

$$2\alpha\beta = \omega\mu\sigma \quad \text{———} (10)$$

We have 2 unknowns α & β.

$$(10) \Rightarrow \quad \beta = \frac{\omega\mu\sigma}{2\alpha}$$

$$(9) \Rightarrow \quad \alpha^2 - \frac{\omega^2\mu^2\sigma^2}{4\alpha^2} = \mu\varepsilon\omega^2$$

$$4\alpha^4 - \omega^2\mu^2\sigma^2 = 4\mu\varepsilon\omega^2\alpha^2$$

$$\cancel{4}\alpha^4 - \cancel{4}\mu\varepsilon\omega^2\alpha^2 - \frac{1}{4}\omega^2\mu^2\sigma^2 = 0$$

$$\alpha^2 = \frac{\mu\varepsilon\omega^2 \pm \sqrt{(\mu\varepsilon\omega^2)^2 + \omega^2\mu^2\sigma^2}}{2}$$

$$\alpha^2 = \frac{\mu\varepsilon\omega^2 \pm \mu\varepsilon\omega^2\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}}{2}$$

$$\alpha^2 = \frac{\mu\varepsilon\omega^2}{2}\left[1 + \sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right]$$

$$\Rightarrow \quad \alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} \quad \text{———} (11)$$

||ly for β, $\alpha = \frac{\omega\mu\sigma}{2\beta}$, We get

& $\cancel{\beta = \frac{\omega\mu\sigma}{2}}$ $\quad \beta^4 + \mu\varepsilon\omega^2\beta^2 - \frac{1}{4}\omega^2\mu^2\sigma^2 = 0$

$$\Rightarrow \quad \beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \quad \text{———} (12)$$

$$\vec{K}^* = k^* \hat{n}$$
$$= (\alpha + i\beta)\hat{n}$$
$$\vec{E} = E_0 e^{i[(\alpha+i\beta)(\hat{n}\cdot\vec{r}) - \omega t]}$$

$$\boxed{\vec{E} = E_0 e^{-\beta(\hat{n}\cdot\vec{r})} e^{i[\alpha(\hat{n}\cdot\vec{r}) - \omega t]}} \quad \text{——— (13)}$$

$$\& \quad \boxed{\vec{B} = B_0 e^{-\beta(\hat{n}\cdot\vec{r})} e^{i[\alpha(\hat{n}\cdot\vec{r}) - \omega t]}} \quad \text{——— (14)}$$

Now Amplitude is exponentially ↓ with space (Not with time).

Decay will be fast or slow, it will depend upon β.

→ β is called Atenuation const. or cofficient.

→ α is propogation cofficient.

→ β depends upon certain properties of medium,

depends upon → ε, μ, σ

If medium is dielectric → $\sigma = 0$ → $\beta \to 0$

then no attenuation in wave.

→ Attenuation in wave is due to σ & conductivity σ is due to free e^-s. {for dielectric → No free e^- so $\sigma = 0$}

→ Here α is like k (wave vector)

Wave Velocity

$$v = \frac{\omega}{\alpha}$$

$$v = \sqrt{\frac{2}{\varepsilon\mu}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]^{-1/2}$$

This is the wave velocity inside the conducting medium.

If we put $\sigma = 0$ then we get expression for dielectric.

$$v = \sqrt{\frac{2}{\varepsilon\mu}} \frac{1}{\sqrt{2}}$$

$$\boxed{v = \frac{1}{\sqrt{\mu\varepsilon}}}$$

Wave velocity is decreased as compare to dielectric.