Electromagnetic Waves inside an Anisotropic Linear Dielectric Medium :-

Generally, for dielectrics, permiability

$$\mu \simeq \mu_0$$

This meadium is Anisotropic w.r.to permitivity ε only.

Maxwell's eqn,   Linear $\rightarrow$ $\vec{D} = \varepsilon \vec{E}$

$\vec{\nabla}\cdot\vec{D} = \rho_f \Rightarrow \boxed{\vec{\nabla}\cdot\vec{D} = 0}$ —(1)   Dielectric $\rightarrow$ $\rho_f = 0$

$\Rightarrow \nabla\cdot(\varepsilon\vec{E}) = 0$ Here ε is not const.

but $\nabla\cdot\vec{E} \neq 0$

$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ —(2)

$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ —(3)

$\vec{\nabla}\times\vec{H} = J_f + \frac{\partial\vec{D}}{\partial t}$ for dielectric $J_f = 0$

$\Rightarrow \boxed{\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t}}$ —(4)

$\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow \vec{K}\cdot\vec{D} = 0$

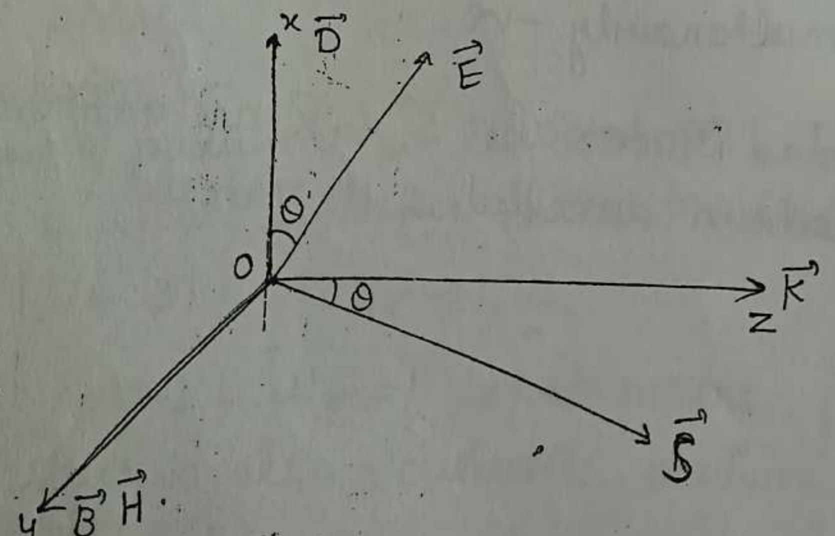
$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{K}\cdot\vec{B} = 0$

i.e. $\vec{B}$ & $\vec{D}$ are $\perp^r$ to wave propogation i.e.

$$\vec{B} \perp \vec{D} \perp \vec{K}$$

$\vec{S} = \vec{E}\times\vec{H}$

& $\vec{B} = \mu_0\vec{H}$

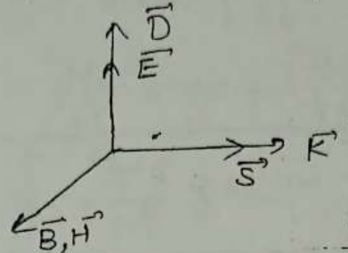


i.e. If medium is Anisotropic then dirn of energy flow is not same as wave prop.

Main Points :-

- EM Waves in Anisotropic medium are transverse w.r.to $\vec{B}$ & $\vec{H}$. Not w.r.to $\vec{E}$ & $\vec{D}$.
- Dirn of Energy flow is not same as the dirn of wave propogation.
- The Electric field is making angle θ with D. Hence $\vec{S}$ is also making angle θ with wave propogation K.

for Isotropic :-



→ Doubly reflecting Systems ($n_x \neq n_y$) [ref. index is not same in all dirn]
are the examples of Anisotropic medium.

Electromag. Waves in Conducting Medium :-

Maxwell's eqn,

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}_f \times \frac{\partial \vec{D}}{\partial t}$$

for a conducting Medium, $\vec{D} = \varepsilon\vec{E}$ & $\vec{B} = \mu\vec{H}$
volume free charge $\rho_f = 0$ & $\boxed{\vec{J}_f = \sigma\vec{E}}$

If we put any free charge in conducting medium then free charge will be on surface. No free charge can reside inside the conducting medium.

$$\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = 0} \quad \text{——(1)}$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0} \quad \text{——(2)}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}} \quad \text{—— (3)}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}} \quad \text{—— (4)}$$

Take curl of eqⁿ (3),

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left[\mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$

$$\nabla^2 E = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 E - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{—— (5)}$$

Take curl of eqⁿ (4),

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \mu\sigma(\vec{\nabla}\times\vec{E}) + \mu\varepsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = \mu\sigma\left(-\frac{\partial\vec{B}}{\partial t}\right) + \mu\varepsilon\left(-\frac{\partial^2\vec{B}}{\partial t^2}\right)$$

$$\boxed{\nabla^2 B - \mu\sigma\frac{\partial\vec{B}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} = 0} \quad \text{—— (6)}$$

eqⁿ (5) & (6) are IInd order differential eqⁿs.

IInd term, of eqⁿ (5) & (6) are damping terms. com

damped harmonic oscillator. $\frac{d^2x}{dt^2} - \omega^2 x =$

Bcoz of this term, amplitude of damped harmonic Oscillator is decaying.

Now, we take wave vector K is a complex quan in the solutions of wave eqⁿ.

$$\vec{E} = E_0 e^{i(\vec{K}^*\cdot\vec{r} - \omega t)}$$

$$\vec{B} = B_0 e^{i(\vec{K}^*\cdot\vec{r} - \omega t)}$$