Electromagnetic waves in Isotropic Linear Dielectric Medium :-

for linear Isotropic (dielectric medium), $\vec{D} = \varepsilon \vec{E}$

$\vec{B} = \mu \vec{H}$

$\vec{D} = \varepsilon \vec{E}$

Maxwell's Eqn

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

for dielectric medium, No free charge, No free charge density

$\rho_f = 0$

$J_f = 0$

$\Rightarrow$ $\nabla\cdot(\varepsilon E) = 0 \Rightarrow$ $\boxed{\vec{\nabla}\cdot\vec{E} = 0}$ ——— (1)

$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ ——— (2)

$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ ——— (3)

$\boxed{\vec{\nabla}\times\vec{B} = \mu\varepsilon\frac{\partial \vec{E}}{\partial t}}$ ——— (4)

Take curl of eqⁿ (3),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\Rightarrow \nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{——— (5)}$$

Take curl of eqⁿ (4),

$$\nabla^2 \vec{B} - \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \quad \text{——— (6)}$$

If we compare these (2) IInd order eqⁿ with original wave eqⁿ, ~~we get~~

$$\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0 \quad \text{——— (7)}$$

We get $v = \frac{1}{\sqrt{\mu\varepsilon}}$ ——— (8)

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

$$v < c$$

So In linear isotropic dielectric, speed of EM wave in that medium is less than speed of light in free space. For Any Transparent Medium,

$$v = \frac{c}{n} \quad \text{——— (9)}$$

$n \rightarrow$ refractive index of medium,
$v \rightarrow$ wave speed

Comparing (8) & (9), We get

$$\boxed{n = \sqrt{\mu_r \varepsilon_r}}$$

So Refractive index of the medium can be calculated by Relative permitivity & Relative permiability.

If medium is Non magnetic $\rightarrow \mu_r = 1$

then $\boxed{n = \sqrt{\varepsilon_r}}$

Now, Maxwell's eqn implies the solutions,

$$\left.\begin{aligned}\vec{K}\cdot\vec{E} &= 0\\ \vec{K}\cdot\vec{B} &= 0\end{aligned}\right\}$$ These eqns implies that →

→ Inside isotropic dielectric, EM waves are transverse wave.

If E & K is given & we have to find B then

$$\boxed{\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}}$$

$$\Rightarrow \cancel{i}(\vec{K}\times\vec{B}) = -\cancel{i}\omega\mu\varepsilon\vec{E}$$

$$\vec{E} = -\frac{1}{\mu\varepsilon\omega}(\vec{K}\times\vec{B})$$

$$\boxed{\vec{E} = -\frac{v^2}{\omega}(\vec{K}\times\vec{B})} \qquad \left\{ v = \frac{1}{\sqrt{\mu\varepsilon}} \right.$$

Now, Magnitude of $\vec{B}$,

$$\boxed{|\vec{B}| = \frac{|\vec{E}|}{v}}$$

where $\frac{\omega}{K} = v$

If wave is travelling in isotropic dielectric medium then $\left[\frac{\omega}{K} = v\right]$.

Conclusion :- $\vec{E} \perp \vec{B} \perp \vec{K}$ i.e. mutually ⊥ to each other.

Energy density :-

Electric energy density $u_e = \frac{\varepsilon}{2}E^2$

Magnetic " " $u_m = \frac{B^2}{2\mu} = \frac{E^2\mu\varepsilon}{2\mu} = \frac{E^2\varepsilon}{2}$

$$u_m = \frac{1}{2}\varepsilon E^2$$

$$\Rightarrow \boxed{u_e = u_m},$$

Total energy density $u = u_e + u_m$

$$\boxed{u = \varepsilon E^2}$$

Poynting Vector :- $\vec{K}\cdot\vec{E} = 0$, $\vec{K}\cdot\vec{B} = 0$

$$\vec{E} = E_0\, e^{i(kz-\omega t)}\,\hat{x}$$

$$\vec{B} = B_0\, e^{i(kz-\omega t)}\,\hat{y}$$

$(B = E\sqrt{\mu\varepsilon} = E/v)$

$$\vec{S} = \vec{E} \times \vec{H}$$
$$= \frac{\vec{E} \times \vec{B}}{\mu} = \frac{1}{\mu v} E^2 = \frac{v}{\mu v^2} E^2$$

$$v^2 = \frac{1}{\mu\varepsilon}$$

$$\boxed{\vec{S} = \varepsilon v E^2 \hat{z} = v u \hat{z}}$$

So Energy flow is in the dir^n of Wave propagation. (dir^n of wave propagation is $\hat{z}$)

On Comparing With free space, We get if we replace

$\mu_0 \longrightarrow \mu$

$\varepsilon_0 \longrightarrow \varepsilon$

$c \longrightarrow v$

then we get, expressions for Isotropic dielectric.

<u>Momentum density</u> $\vec{P} = \mu\varepsilon\vec{S} = \frac{u}{v}\hat{z}$

<u>Wave Impedence</u> $Z = \left|\frac{E}{H}\right| = \sqrt{\frac{\mu}{\varepsilon}}$

$$Z = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

If $\left.\begin{matrix}\mu_r \geq 1 \\ \varepsilon_r \geq 1\end{matrix}\right\}$ $Z = 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}$ = Real Value

If Z is Real that means $\vec{E}$ & $\vec{B}$ are vibrating in same phase.

<u>Conclusion</u> :-

→ speed of medium v is less than c. & → $n = \sqrt{\mu_r \varepsilon_r}$

→ for a linear isotropic dielectric medium

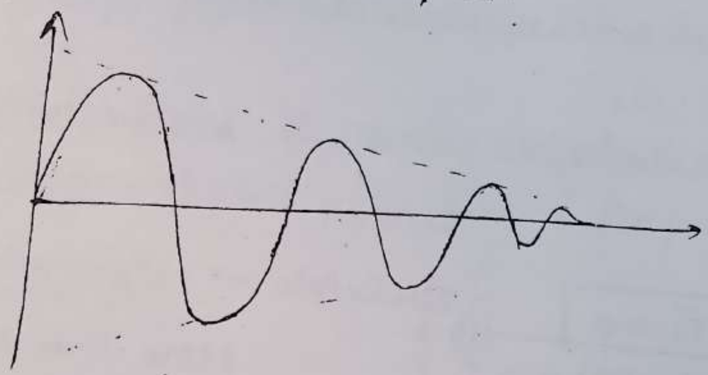
$|\mu_r \geq 1$ & $\varepsilon_r \geq 1$

(i) If $\varepsilon_r = 1$, $\mu_r = 1$ that means free space
free space is also a dielectric medium of dielectric constant one.

(ii) If $\varepsilon_r > 1$, $\mu_r > 1$ then n is real
then sol^n of EM will be Oscillatory

(iii) If $\varepsilon_r < 0$ (-ve), $\mu_r \geq 1$ $\Rightarrow n \rightarrow$ imaginary

If n is imaginary or complex, then solution of EM wave will be damped.



If $\varepsilon_r \geq 0$, $\mu_r < 0$, Again $sol^n \rightarrow$ damped (Inside the conducting medium)

- In all these cases, dir^n of $\underset{\text{Energy flow}}{\vec{S}}$ & $\underset{\text{wave prop.}}{\vec{K}}$ will be same.

Here ~~If~~, ε_r & μ_r are not the fu^n of position then medium will be isotropic.

<u>Note</u> :- If ε_r & μ_r are fu^n of position then dir^n of $\vec{S}$ & $\vec{K}$ are different.

<u>Case (iv)</u> :- If ε_r & μ_r are simultanously negative.

then $\vec{S}$ & $\vec{K}$ are antiparallel. $\left\{ \begin{array}{l} \mu = \mu_0 \mu_r \\ \varepsilon = \varepsilon_0 \varepsilon_r \end{array} \right.$

No medium exist in nature, in which both ε_r & μ_r are simultanously -ve.

For Left Handed Material, $\vec{S}$ & $\vec{K}$ are antiparallel.

e. This type of medium is called L.H. material.