Momentum Density of EM Wave :-

Total mom. per unit volume called Mom. density. denoted by $\vec{p}$

In free Space, $\vec{p} = \mu_0 \varepsilon_0 \vec{S}$

$$\boxed{\vec{p} = \frac{\vec{S}}{c^2} = \frac{u}{c}\hat{z}} \qquad (\vec{S} = cu)$$

This is called Electromag. mom. density.

Average Value of these quantities :-

We have, Energy density $u = \varepsilon_0 E^2$

Poynting vector, $\vec{S} = c\,\varepsilon_0 E^2 \hat{z}$

Momentum density $p = \frac{\varepsilon_0 E^2}{c}\hat{z}$

Now, we calculate average energy density, Average poynting vector & average mom. density over a cycle.

We know, the solutions are

$$\vec{E} = E_0 e^{i(kz-\omega t)}\hat{x}$$

$$\vec{B} = B_0 e^{i(kz-\omega t)}\hat{y}$$

We can write,

$$\vec{E} = E_0\left[\underbrace{\cos(kz-\omega t)}_{\text{real Part}} + i\underbrace{\sin(kz-\omega t)}_{\text{imaginary Part}}\right]\hat{x}$$

Only Real part carries the energy.

$$E^2 = \vec{E}\cdot\vec{E} = E_0^2\cos^2(kz-\omega t)$$

Average Value of energy density

$$\langle u\rangle = \varepsilon_0 E_0^2 \langle \cos^2(kz-\omega t)\rangle$$

Average value of $\cos^2$ over a cycle of 2π gives $\frac{1}{2}$.

so $\boxed{\langle u\rangle = \frac{1}{2}\varepsilon_0 E_0^2}$ → we can also write it in terms of mag. field (B_0) → amp of mag. field

E_0 → amplitude

This is the combination of electric & magnetic energy density

So seperately,

$$\langle u_e \rangle = \frac{1}{4}\varepsilon_0 E_0^2$$

$$\langle u_m \rangle = \frac{1}{4}\varepsilon_0 E_0^2$$

Relation b/w amplitude of elec. & mag. field, $B_0 = \frac{E_0}{c}$

$$\Rightarrow E_0 = B_0 c$$

$$\Rightarrow E_0^2 = B_0^2 c^2$$

$$\Rightarrow E_0^2 = \frac{B_0^2}{\mu_0 \varepsilon_0}$$

So $\boxed{\langle u \rangle = \frac{1}{\cancel{2}4}\varepsilon_0 E_0^2 = \frac{B_0^2}{4\mu_0}}$

Average Value of poynting vector also called Intensity
it is the average energy per unit area per unit time associated with the EM wave.

$$\langle \vec{S} \rangle = I$$

e.g. An EM wave of given intensity, incident on $1\ m^2$ area for 10 sec then calculate energy transfered to the surface.

$$I = 2\ J/m^2 sec$$

energy / unit area / unit time

so Energy $= 2 \times 10 = 20$ Joule

Now,

$$\langle \vec{S} \rangle \equiv I \text{ (intensity)}$$

$$= c \langle u \rangle \hat{z}$$

$$\boxed{\langle \vec{S} \rangle = \frac{1}{2}\varepsilon_0 c E_0^2 \hat{z}}$$ → This much intensity is transfe to the surface.

This can be in another form (in terms of mag. field)

Average Value of Mom. density

Mom. density → mom. / unit volume.

If Mom. density of EM wave is given ~~for~~ a unit volume & calculate mom. transfer to the given volume then

$$\langle \vec{p} \rangle = \frac{1}{2}\frac{1}{c}\, \varepsilon_0 \langle E_0^2 \rangle$$

$$\boxed{\langle \vec{p} \rangle = \frac{\varepsilon_0}{2}\frac{E_0^2}{c}}$$

This much avg. mom./unit volume will transfer to the volume.

Radiation pressure (P)

Pressure → force per unit area

$$P = \frac{F}{A}$$

If a surface is made of atoms contain charges & e⁻ ~~eleve~~ revolve in the orbites. If EM wave incident on the surface of atom. Then the force applied per unit area is called the Radiation pressure.

force → Rate of change of momentum
i.e. mom. per unit time

We know the mom. density which is mom./unit volum.
i.e. If Mom. density × volume = ~~force~~ mom.
& ~~force~~ mom. per unit ~~area~~ time gives force & force/unit area gives pressure.

$$\langle \vec{p} \rangle = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{c}$$

Let Mom. transfered in time Δt is Δp

$$\Delta p = \langle p \rangle A c \Delta t$$

{ If EM wave travell with speed of light then in time Δt it will travell distance $\underline{c\Delta t}$. }

Now force

$$F = \frac{\Delta p}{\Delta t} = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{\cancel{c}} \times \cancel{c} A$$

time = $\frac{\text{distance}}{\text{speed}}$ = $\frac{\cancel{c}\Delta t}{\cancel{c}}$

Pressure $P = \frac{F}{A}$ ⇒ $\boxed{P = \frac{c}{2c}\varepsilon_0 E_0^2 = \frac{I}{c}}$

This much pressure will be on the surface due to electromag. wave.
So Radiation pressure is Intensity devided by speed of light EM wave.

→ This is the radiation pressure if the ~~master~~ medium is perfecetely Absorbing ($P = \frac{I}{c}$)

→ If surface is perfeectely reflecting then pressure will be doubled. $\boxed{P = \frac{2I}{c}}$

Note:-

- Metal reflects EM Waves (Mirrors are reflectors) bcoz metal contains large no. of free e^-s. So free e^-s are responsible for this reflection.

→ If we incident EM wave on a mirror then free e^- starts to vibrate with the freq of EM wave then it will oscillate & emit radiation. (i.e. e^- Reradiate the EM wave) in this phenomenon No time lag.

→ When EM wave incident on surface then (One) pressure produce. & when it reradiate then again there will be pressure so pressure will be doubled.

→ If for perfeetelly Absorbing medium, when wave re-radiate then there will be no pressure. So in this case there is single pressure.

Microwaves & EM wave they can travell with Insulators.

Wave Impedence of free Space :-

It is denoted by Z_0. & defined as

$$Z_0 = \left|\frac{E_0}{H_0}\right|$$

$E_0 \rightarrow$ Amp of E-field

$H_0 \rightarrow$ Amp of mag. field intensity.

$$B_0 = \mu H_0 \Rightarrow H_0 = \frac{B_0}{\mu}$$

So $Z_0 = \left|\frac{E_0}{H_0}\right| = \frac{\mu_0 E_0}{B_0} = \mu_0 c = \mu_0 \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\boxed{Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120 \cdot \pi}$$ This is the impedence of free space.

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi = 377\ \Omega$$

In free space, Z_0 is a real quantity.

Hence $\vec{E}$ & $\vec{B}$ vibrates in <u>same phase</u>. (in free space)

$$\vec{E} = E_0 \cos(kz - \omega t)\, \hat{x}$$

$$\vec{B} = B_0 \cos(kz - \omega t)\, \hat{y}$$

There is No phase diff.

<u>Important points of free space</u> :-

→ EM waves are transvers in free space.

i.e. $E \perp K$ & $B \perp K$

$E \perp B \perp K$

They are moving with speed of light c.

→ $\vec{E}$ & $\vec{B}$ are vibrating in same phase.

→ Same energy lies in $\vec{E}$ & $\vec{B}$ field.