Unit VI: Interference

Interference of light

The phenomenon of redistribution of energy due to super position of light waves from two coherent sources is called interference.

Constructive Interference: In constructive interference the amplitude of the resultant wave is greater than that of either individual wave.

Destructive Interference: In destructive interference the amplitude of the resultant wave is less than that of either individual wave.

Interference can be obtained through two methods

1. Division of Wave front
2. Division of Amplitude

In **Division of Wave front**, the coherent sources are obtained by dividing the wavefront, originating from a common source, by employing pinholes, narrow slits, mirrors, biprisms or lenses. This class of interference requires essentially a point source or a narrow slit source. Examples of interference by division of wavefront are Young's Double slit experiment, the Fresnel biprism, Fresnel mirrors, Lloyd's mirror, etc

In **Division of Amplitude**, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. Thus we have coherent beams produced by division of amplitude. These beams travel different paths and are finally brought together to produce interference. The effects resulting from the superposition of two beams are referred to as two beam interference and those resulting from superposition of more than two beams are referred to as multiple beam interference. The interference in thin films, Newton's rings, and Michelson's interferometer are examples of two beam interference and Fabry-Perot's interferometer is an example of multiple beam interference.

Why single source is required for interference?

If we use two conventional light sources (like two sodium lamps) illuminating two pinholes (see Fig. 14.5), we will not observe any interference pattern on the screen. This can be understood from the following reasoning: In a conventional light source, light comes from a large number of independent atoms; each atom emitting light for about 10^{-10} sec, i.e., light emitted by an atom is essentially a pulse lasting for only 10^{-10} seconds* Even if the atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases.

Consequently, light coming out from the holes S_1 and S_2 will have a fixed phase relationship for a period of about 10^{-10} seconds, hence the interference pattern will keep on changing every billionth of a second. The eye can notice intensity changes which last at least for a tenth of a second and hence we will observe a uniform intensity over the screen. However, if we have a camera whose time of shutter opening can be made less than 10^{-10} seconds then the film will record an interference pattern**. We summarise the above results by noting that light beams from two independent sources do not have any fixed phase relationship***, as such they do not produce any stationary interference pattern.

Young's double slit experiment

The experiment

Thomas Young in 1801 devised an ingenious but simple method to lock the phase relationship between the two sources. The trick lies in the division of a single wavefront into two; these two split wavefronts act as if they emanated from two sources having a fixed phase relationship and, therefore, when these two waves were allowed to interfere, a stationary interference pattern was obtained. In the actual experiment, a light source illuminates the pinhole S (see Fig. 14.6). Light diverging from this pinhole fell on a barrier which contained two pinholes S_1 and S_2 which were very close to one another and were located equidistant from S. Spherical waves emanating from S_1 and S_2 (see Fig. 14.7) were coherent and on the screen beautiful interference fringes were obtained. In order to show that this was indeed an interference effect, Young showed that the fringes on the screen disappear when S_1 (or S_2) is covered up. Young explained the interference pattern by considering the principle of superposition, and by measuring the distance between the fringes he calculated the wavelength. Figure 14.7 shows the section of the wavefront on the plane containing S, S_1 and S_2 (which is the $x - z$ plane).

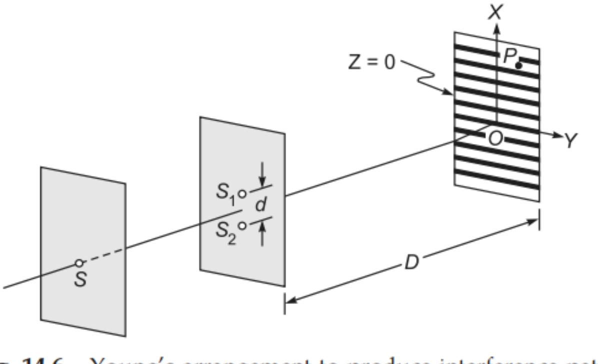


Fig. 14.6 Young's arrangement to produce interference pattern.

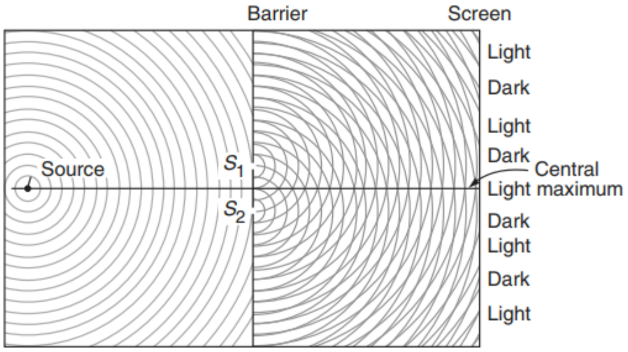


Fig. 14.7 Sections of the spherical wavefronts emanating from S, S_1 and S_2 (Adapted from Ref. 14.7; used with permission).

The Interference Pattern

In this section, we will first obtain an expression for the fringe width and then we will show that the fringes are strictly hyperbolic.

Let S_1 and S_2 represent the two pinholes of the Young's double hole arrangement. We would determine the positions of maxima and minima on the line LL' which is parallel to the x axis and lies in the plane containing S, S_1 and S_2 (see Fig. 14.8). We will show that the interference pattern (around the point O) consists approximately of a series of dark and bright lines perpendicular to the plane of Fig. 14.8; O being the foot of the perpendicular from the point S on the screen.

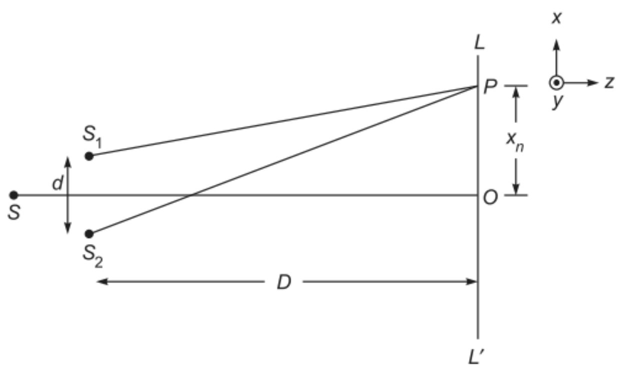


Fig. 14.8 Arrangement for producing Young's interference pattern.

For an arbitrary point P (on the line LL') to correspond to a maximum we must have

$$S_2P - S_1P = n\lambda;\ n = 0, 1, 2, 3, \ldots \qquad \textbf{(14.17)}$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x_n - \frac{d}{2}\right)^2\right] = 2x_nd \qquad \textbf{(14.18)}$$

where $S_1S_2 = d$ and $OP = x_n$. Thus,

$$S_2P - S_1P = \frac{2x_nd}{S_2P + S_1P} \approx \frac{x_nd}{D} \qquad \textbf{(14.19)}$$

where in the last step we have replaced $S_2P + S_1P$ by $2D$ which will be valid when $D >>> d, x_n$. For example, for $d = 0.02$ cm, $D = 50$ cm, and $OP = 0.5$ cm (which corresponds to typical values in a light interference experiment) we will have

$$S_2P + S_1P = \sqrt{(50)^2 + (0.51)^2} + \sqrt{(50)^2 + (0.49)^2}$$
$$= 100.005 \text{ cm}$$

Thus, if we replace $S_2P + S_1P$ by $2D$, the error involved is about 0.005%. Using Eqs. (14.17) and (14.19) we obtain

$$x_n = \frac{n\lambda D}{d} \qquad \textbf{(14.20)}$$

Thus, the bright (and dark) fringes are equally spaced and the distance between two consecutive bright (or dark) fringes is given by

$$\beta = x_{n+1} - x_n = \frac{\lambda D}{d} \qquad \textbf{(14.21)}$$

which is the expression for the fringe width.

We will next determine the shape of the interference pattern on the screen LL' and show that the fringes are a set of hyperbolae. We assume the origin to be at the point O and the z-axis to be perpendicular to the plane of the screen LL' as shown in Fig. 14.8. The screen LL' corresponds to the plane $z = 0$; thus the coordinates of an arbitrary point P on the screen will be $(x, y, 0)$. The coordinates of the point S_1 and S_2 would be $\left(+\frac{d}{2}, 0, -D\right)$ and $\left(-\frac{d}{2}, 0, -D\right)$, respectively. Thus,

$$S_2P - S_1P = \left[\left(x + \frac{d}{2}\right)^2 + y^2 + D^2\right]^{1/2} - \left[\left(x - \frac{d}{2}\right)^2 + y^2 + D^2\right]^{1/2}$$
$$= \Delta \text{ (say)}$$

or

$$\left(x + \frac{d}{2}\right)^2 + y^2 + D^2 = \left\{\Delta + \left[\left(x - \frac{d}{2}\right)^2 + y^2 + D^2\right]^{1/2}\right\}^2$$

Simple manipulations will give us

$$(d^2 - \Delta^2)x^2 - \Delta^2y^2 = \Delta^2\left[D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]$$

For $\Delta = 0$ (zero path difference) we must have $x = 0$ which implies that the central (bright) fringe is along the y-axis; this is rigorously true. In general, the above equation can always be written in the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \textbf{(14.22)}$$

where

$$a^2 = \frac{\Delta^2}{(d^2 - \Delta^2)}\left[D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]$$

and $$b^2 = D^2 + \frac{1}{4}(d^2 - \Delta^2) \quad \textbf{(14.23)}$$

Equation (14.22) represents a hyperbola. On rearranging, we get

$$x = \pm\left(\frac{\Delta^2}{d^2 - \Delta^2}\right)^{1/2}\left[y^2 + D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]^{1/2} \quad \textbf{(14.24)}$$

Obviously for $y^2 << D^2$, we may neglect y^2 inside the square brackets and the loci are straight lines parallel to the y-axis. Thus, we obtain straight line fringes on the screen. We must remember that we had assumed point sources and we obtained straight line fringes. It is easy to see that if we had slits instead of point sources, each pair of points would have produced the same straight line fringes which would have overlapped with each other–thus we would again obtain straight line fringes. The fringes so produced are said to be non-localized; they can be photographed by just placing a film on the screen; they can also be seen through an eye-piece.

The Intensity Distribution

Let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the electric fields produced at the point P by S_1 and S_2 respectively (see Fig. 14.8). The electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ will, in general, have different directions and different magnitudes. However, if the distances S_1P and S_2P are very large in comparison to the distance S_1S_2, the two fields will almost be in the same direction. Thus, we may write

$$\left.\begin{aligned}\mathbf{E}_1 &= \hat{\mathbf{i}}E_{01}\cos\left(\frac{2\pi}{\lambda}S_1P - \omega t\right)\\ \text{and}\quad \mathbf{E}_2 &= \hat{\mathbf{i}}E_{02}\cos\left(\frac{2\pi}{\lambda}S_2P - \omega t\right)\end{aligned}\right\} \quad \textbf{(14.25)}$$

where $\hat{\mathbf{i}}$ represents the unit vector along the direction of either of the electric fields. The resultant field will be given by

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2\\ &= \hat{\mathbf{i}}\left[E_{01}\cos\left(\frac{2\pi}{\lambda}S_1P - \omega t\right)\right.\\ &\quad \left. + E_{02}\cos\left(\frac{2\pi}{\lambda}S_2P - \omega t\right)\right]\end{aligned} \quad \textbf{(14.26)}$$

The intensity (I) will be proportional to the square of the electric field and will be given by

$$I = KE^2 \quad \textbf{(14.27)}$$

or

$$\begin{aligned}I = K\Bigg[&E_{01}^2\cos^2\left(\frac{2\pi}{\lambda}S_1P - \omega t\right) +\\ &E_{02}^2\cos^2\left(\frac{2\pi}{\lambda}S_2P - \omega t\right) +\\ &E_{01}E_{02}\left\{\cos\left[\frac{2\pi}{\lambda}(S_2P - S_1P)\right] +\right.\\ &\left.\cos\left[2\omega t - \frac{2\pi}{\lambda}(S_2P + S_1P)\right]\right\}\Bigg]\end{aligned} \quad \textbf{(14.28)}$$

where K is a proportionality constant.* For an optical beam the frequency is very large ($\omega \approx 10^{15}$ sec^{-1}) and all the terms depending on ωt will vary with extreme rapidity (10^{15} times in a second); consequently, any detector would record an average value of various quantities. Now,

$$\begin{aligned}\langle\cos^2(\omega t - \theta)\rangle &= \frac{1}{2\tau}\int_{-\tau}^{+\tau}\frac{1 + \cos[2(\omega t - \theta)]}{2}dt\\ &= \frac{1}{2} + \frac{1}{16\pi}\frac{T}{\tau}\left\{\left[\sin 2(\omega t - \theta)\right]_{-\tau}^{+\tau}\right\}\end{aligned}$$

where $T = \frac{2\pi}{\omega}$ ($\approx 2\pi \times 10^{-15}$ sec for an optical beam). For any practical detector** $\frac{T}{\tau} <<< 1$ and since the quantity between the curly brackets will always be between -2 and $+2$, we may write

$$\langle\cos^2(\omega t - \theta)\rangle \approx \frac{1}{2} \quad \textbf{(14.29)}$$

The factor $\cos(2\omega t - \phi)$ will oscillate between $+1$ and -1 and its average will be zero as can indeed be shown mathematically. Thus the intensity, that a detector will record, will be given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \quad \textbf{(14.30)}$$

where

$$\delta = \frac{2\pi}{\lambda}(S_2P - S_1P) \quad \textbf{(14.31)}$$

represents the phase difference between the displacements reaching the point P from S_1 and S_2. Further

$$I_1 = \frac{1}{2}KE_{01}^2$$

represents the intensity produced by the source S_1 if no light from S_2 is allowed to fall on the screen; similarly $I_2 = \frac{1}{2}KE_{02}^2$ represents the intensity produced by the source S_2 if no light from S_1 is allowed to fall on the screen. From Eq. (14.30) we may deduce the following:

(a) The maximum and minimum values of $\cos\delta$ are $+1$ and -1, respectively; as such the maximum and minimum values of I are given by

$$\left.\begin{aligned}I_{max} &= (\sqrt{I_1} + \sqrt{I_2})^2\\ \text{and}\quad I_{min} &= (\sqrt{I_1} - \sqrt{I_2})^2\end{aligned}\right\} \quad \textbf{(14.32)}$$

The maximum intensity occurs when

$$\delta = 2n\pi, \quad n = 0, 1, 2, \ldots$$

or

$$S_2P \sim S_1P = n\lambda,$$

and the minimum intensity occurs when

$$\delta = (2n + 1)\pi;\ n = 0, 1, 2, \ldots$$

or

$$S_2P \sim S_1P = \left(n + \frac{1}{2}\right)\lambda$$

Notice that when $I_1 = I_2$, the intensity minimum is zero. In general, $I_1 \neq I_2$ and the minimum intensity is not zero.

(b) If the holes S_1 and S_2 are illuminated by different light sources (see Fig. 14.5), then the phase difference δ will remain constant for about 10^{-10} sec (see discussion in Sec. 14.3) and thus δ would also vary with time*** in a random way. If we now carry out the averaging over time scales which are of the order of 10^{-8} sec, then time scales which are of the order of 10^{-8} sec, then

$$\langle \cos \delta \rangle = 0$$

and we obtain

$$I = I_1 + I_2$$

Thus, for two incoherent sources, the resultant intensity is the sum of the intensities produced by each one of the sources independently and no interference pattern is observed.

(c) In the arrangement shown in Fig. 14.6, if the distances S_1P and S_2P are large in comparison to d, then

$$I_1 \approx I_2 = I_0 \text{ (say)}$$

and

$$I = 2I_0 + 2I_0 \cos \delta = 4I_0 \cos^2 \frac{\delta}{2} \qquad \textbf{(14.33)}$$

The intensity distribution (which is often termed as the $\cos^2$ pattern) is shown in Fig. 14.9.

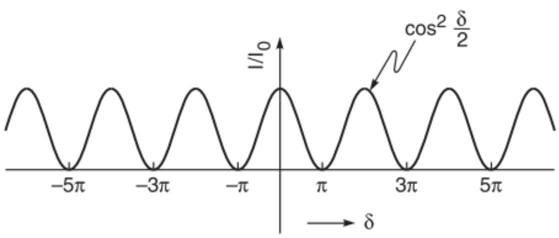


Fig. 14.9 The variation of intensity with δ.

Fresnel's Biprism experiment

From the Young's Double slit experiment we obtained the fringe width β of interference pattern which is the distance between successive bright fringes or successive dark fringes is determined by the following equation

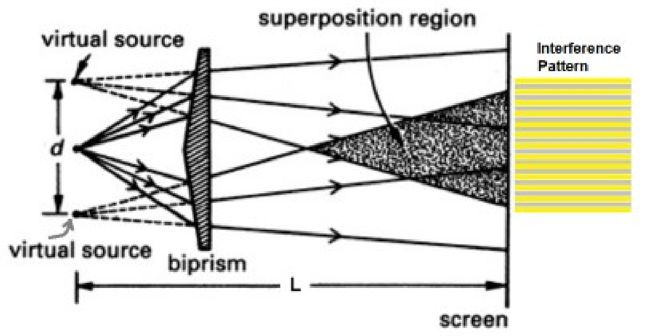
$$\beta = \frac{\lambda D}{d} \text{ ------------- }(1)$$

where λ is the wavelength of light , D is the distance between double slits and screen and d is the distance between slits S_1 and S_2 .

Wavelength λ is determined from experimental value of β and from known values of D and d .

Drawback in Young's Double slit experiment is the assumption of monochromatic sources S_1 and S_2 as point sources, because it is very difficult to get narrow slits that act as point source. If we make the slit very narrow, intensity of light decreases and interference pattern will be very faint that leads to difficulty in measurement of fringe-width.

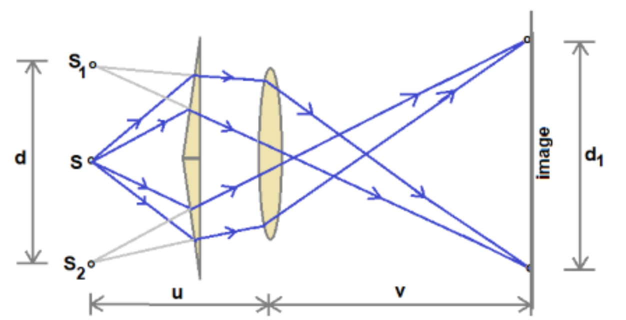
This difficulty is overcome by Fresnel's Biprism experiment as explained below.

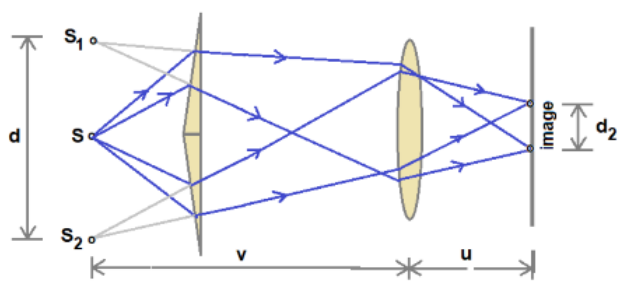


Biprism is formed by merging two narrow prisms at their base. A narrow slit is paced before biprism so that slit is in line with centre of biprism. A single wavefront emerging from the narrow slit is getting refracted.

After refraction , light rays merge together on a screen to form interference pattern as shown in figure. Refracted light rays are acting in this process such that as if they are coming from two virtual point sources of light as shown in figure. This experimental setup is identical like Young's Double slit experiment where we get interference pattern from two point sources of light that are in same phase. If we know the distance d between virtual sources, we use the fringe width equation as given in Young's Double slit experiment and determine the wavelength of monochromatic light .

To determine the distance d between virtual sources we use images formed by convex lens as shown below





Let us get the image of virtual sources by placing a convex lens as shown above.

From lens equation we get the relation between the distance u and v as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ --------- }(2)$$

Let us get the image of virtual sources by placing a convex lens as shown above.

From lens equation we get the relation between the distance u and v as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ ------------ }(2)$$

where v is lens-to-image distance and u is lens-to-object distance

The relation of magnification of image d_1 is

$$m_1 = \frac{d_1}{d} = -\frac{v}{u} \text{ ------------ }(3)$$

When $v > u$, we get a magnified image.

In the lens equation (2) , u and v are interchangeable. Hence if we position the convex lens at a distance v from object , we get a diminished image at a distance v

Now the relation of magnification of the new image d_2 is

$$m_2 = \frac{d_2}{d} = -\frac{u}{v} \text{ ------------ }(4)$$

By multiplying eqn. (3) and eqn. (4) , we get

$$\frac{d_1}{d} \times \frac{d_2}{d} = \left(-\frac{v}{u}\right) \times \left(-\frac{u}{v}\right) = 1$$

$$d^2 = d_1 d_2$$

Thus,

$$d = \sqrt{d_1 d_2}$$

Hence , using the distance d between virtual sources in eqn. (1) and from the measured values of fringe-width β it is possible to determine the wavelength λ of monochromatic light .

Phase change on reflection: Stokes' treatment

We will now investigate the reflection of light at an interface between two media using the principle of optical reversibility. According to this principle, in the absence of any absorption, a light ray that is reflected or refracted will retrace its original path if its direction is reversed.*

Consider a light ray incident on an interface of two media of refractive indices n_1 and n_2 as shown in Fig. 14.22(a). Let the amplitude reflection and transmission coefficients be r_1 and t_1, respectively. Thus, if the amplitude of the incident ray is a, then the amplitudes of the reflected and refracted rays would be ar_1 and at_1, respectively.

We now reverse the rays and we consider a ray of amplitude at_1 incident on medium 1 and a ray of amplitude ar_1 incident on medium 2 as shown in Fig. 14.22(b). The ray of amplitude at_1 will give rise to a reflected ray of amplitude at_1r_2 and a transmitted ray of amplitude at_1t_2 where r_2 and t_2 are the amplitude reflection and transmission coefficients when a ray is incident from medium 2 on medium 1. Similarly, the ray of amplitude ar_1 will give rise to a ray of amplitude ar_1^2 and a refracted ray of amplitude ar_1t_1. According to the principle of optical reversibility the two rays of amplitudes ar_1^2 and at_1t_2 must combine to give the incident ray of Fig. 14.22(a); thus,

$$ar_1^2 + at_1t_2 = a$$

or

$$t_1t_2 = 1 - r_1^2 \qquad \textbf{(14.38)}$$

Further, the two rays of amplitudes at_1r_2 and ar_1t_1 must cancel each other, i.e.,

$$at_1r_2 + ar_1t_1 = 0$$

or

$$r_2 = -r_1 \qquad \textbf{(14.39)}$$

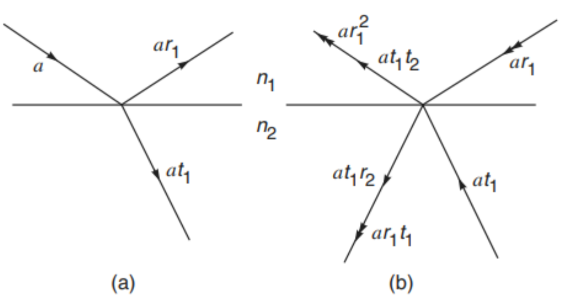


Fig. 14.22 (a) A ray traveling in a medium of refractive index n_1 incident on a medium of refractive index n_2. (b) Rays of amplitude ar_1 and at_1 incident on a medium of refractive index n_1.

Since we know from the Lloyd's mirror experiment that an abrupt phase change of π occurs when light gets reflected by a denser medium, we may infer from Eq. (14.39) that no such abrupt phase change occurs when light gets reflected by a rarer medium. This is indeed borne out by experiments. Equations (14.38) and (14.39) are known as **Stokes' relations**.

Considering Fig:14.22 (a), the reflected ray of amplitude ar_1 undergoes a π phase shift to that of the primary incident ray of amplitude a, while the transmitted ray of amplitude at_1 has the same phase as the incident ray. Now in Fig:14.22 (b), the ray with amplitude ar_1t_1 will be in phase with the ray of amplitude ar_1. If we consider the reflected ray of amplitude at_1r_2 will undergo a π phase shift on reflection from medium I from that of the ray of amplitude at_1, the two rays of amplitudes ar_1t_1 and at_1r_2 will be in same phase which contradicts Eq. 14.39. Thus there is no phase change if light coming from a denser medium is reflected at an interface of a rarer medium (i.e light incident on medium I from medium II in case $n_1 < n_2$).

Interference by plane parallel film when illuminated by a plane wave

Normal incidence:

If a plane wave is incident normally on a thin* film of uniform thickness d (see Fig. 15.1) then the waves reflected from the upper surface interfere with the waves reflected from the lower surface. In this section, we will study this interference pattern. In order to observe the interference pattern without obstructing the incident beam, we use a partially reflecting

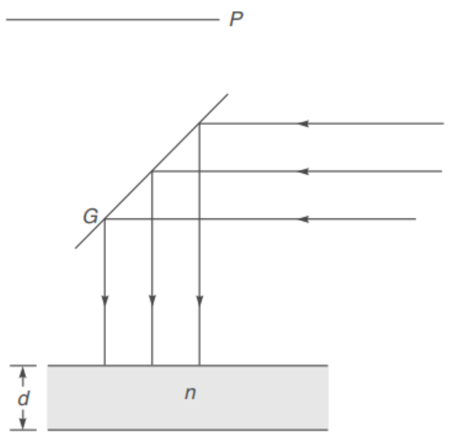


Fig. 15.1 The normal incidence of a parallel beam of light on a thin film of refractive index n and thickness d. G denotes a partially reflecting plate and P represents a photographic plate.

plate G as shown in Fig. 15.1. Such an arrangement also enables us to eliminate the direct beam from reaching the photographic plate P (or the eye). The plane wave may be produced by placing an illuminated pinhole at the focal point of a corrected lens; alternatively, it may just be a beam coming out of a laser.

Let the solid and the dashed lines in Fig. 15.2 represent the positions of the crests** (at any particular instant of time) corresponding to the waves reflected from the upper and lower surfaces of the film, respectively.*** Clearly, the wave reflected from the lower surface of the film traverses an additional optical path of $2nd$, where n represents the refractive index of the material of the film. Further, if the film is placed in air, then the wave reflected from the upper surface of the film will undergo a sudden change in phase of π (see Sec. 14.12) and as such the conditions for destructive or constructive interference will be given by

$$2nd = m\lambda \quad \text{destructive interference} \qquad \textbf{(15.1a)}$$

$$= \left(m + \frac{1}{2}\right)\lambda \quad \text{constructive interference} \qquad \textbf{(15.1b)}$$

where $m = 0, 1, 2, \ldots$ and λ represents the free space wavelength.

Thus, if we place a photographic plate at P (see Fig. 15.1), then the plate will receive uniform illumination; it will be dark when $2nd = m\lambda$ and bright when $2nd = \left(m - \frac{1}{2}\right)\lambda$; $m = 0, 1, 2, \ldots$ Instead of placing the photographic plate, if we try to view the film (from the top) with naked eye, then the film will appear to be uniformly illuminated.

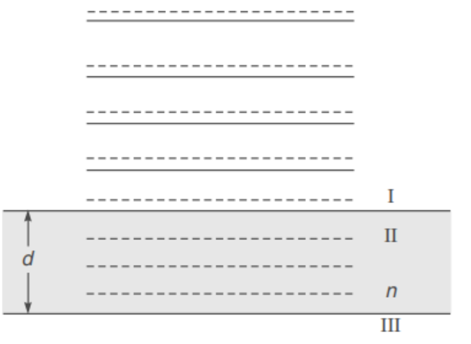


Fig. 15.2 The solid and the dashed lines represent the crests of the waves reflected from the upper surface and from the lower surface of the thin film. Notice that the distance between the consecutive crests inside the film is less than the corresponding distance in medium I.

It may be noted that the amplitudes of the waves reflected from the upper and lower surfaces will, in general, be slightly different; and as such the interference will not be completely destructive. However, with appropriate choice of the refractive indices of media II and III, the two amplitudes can be made very nearly equal (see Example 15.1).

For an air film between two glass plates (see Fig. 15.3) no phase change will occur on reflection at the glass-air interface, but a phase change of π will occur on reflection at the air-glass interface and the conditions for maxima and minima will remain the same. On the other hand, if the I medium is crown glass ($n = 1.52$), the II medium is an oil of refractive index 1.60 and the III medium is flint glass ($n = 1.66$) then a phase change of π will occur at both the reflections and the conditions for maxima and minima would be

$$2nd = \left(m + \frac{1}{2}\right)\lambda \quad \text{minima} \qquad \textbf{(15.2a)}$$

$$= m\lambda \quad \text{maxima} \qquad \textbf{(15.2b)}$$

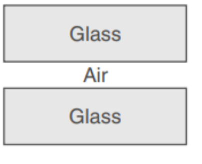


Fig. 15.3 Thin film of air formed between two glass plates.

In general, whenever the refractive index of the II medium lies in between the refractive indices of the I and the III media, then the conditions of maxima and minima would be given by Eqs. (15.2a) and (15.2b).

Oblique incidence:

We next consider the oblique incidence of the plane wave on the thin film (see Fig. 15.4). Once again, the wave reflected from the upper surface of the film interferes with the wave reflected from the lower surface of the film. The latter traverses an additional optical path Δ, which is given by (see Fig. 15.5):

$$\Delta = n_2(BD + DF) - n_1 BC \quad \textbf{(15.3)}$$

where C is the foot of the perpendicular from the point F on BG.

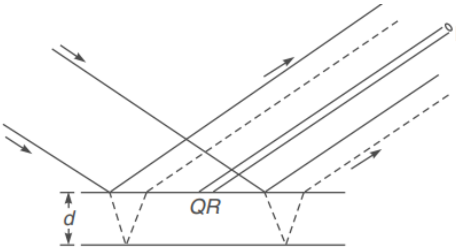


Fig. 15.4 The oblique incidence of a plane wave on a thin film. The solid and dashed lines denote the boundary of the wave reflected from the upper surface and from the lower surface of the film. The eye E receives the light reflected from the region QR.

Let θ and θ' denote the angles of incidence and refraction respectively. We drop a perpendicular BJ from the point B on the lower surface LL' and extend BJ and FD to the point B' where they meet (see Fig. 15.5). Clearly,

$$\angle JBD = \angle BDN = \angle NDF = \theta'$$

where N is the foot of the perpendicular drawn from the point D on BF. Now,

$$\angle BDJ = \frac{\pi}{2} - \theta'$$

and $$\angle B'DJ = \pi - \left[\left(\frac{\pi}{2} - \theta'\right) + \theta' + \theta'\right] = \frac{\pi}{2} - \theta'$$

Thus, $BD = BD'$ and $BJ = JB' = d$

or $BD + DF = B'D + DF = B'F$

Hence, $$\Delta = n_2 B'F - n_1 BC \quad \textbf{(15.7)}$$

Now, $\angle CFB = \angle CBX = \theta$

$$BC = BF \sin\theta = \frac{KF}{\sin\theta'} \sin\theta = \frac{n_2}{n_1} KF \quad \textbf{(15.8)}$$

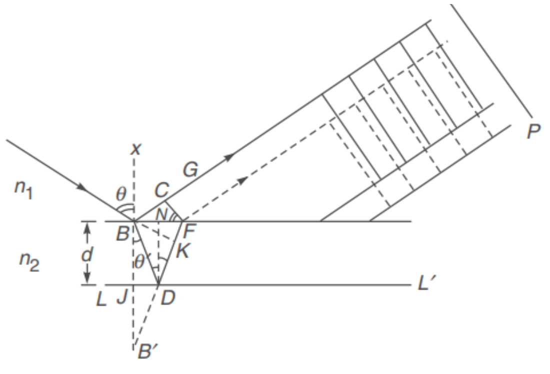


Fig. 15.5 Calculation of the optical path difference between the waves reflected from the upper surface of the film and from the lower surface of the film. The solid and the dashed lines represent the corresponding positions of the crests. P denotes a photographic plate.

where K is the foot of the perpendicular from B on $B'F$. Substituting the above expression for BC in Eq. (15.7), we get

$$\Delta = n_2 B'F - n_2 KF = n_2 B'K$$

or $$\Delta = 2 n_2 d \cos\theta' \quad \textbf{(15.9)}$$

which is known as the **cosine law**.

For a film placed in air, a phase change of π will occur when reflection takes place at the point B and as such, the conditions of destructive and constructive interference would be given by

$$\Delta = 2 n_2 d \cos\theta' = m\lambda \qquad \text{minima} \quad \textbf{(15.5a)}$$

$$= \left(m + \frac{1}{2}\right)\lambda \qquad \text{maxima} \quad \textbf{(15.5b)}$$

If we place a photographic plate at P (see Fig. 15.5) it will receive uniform illumination; if we try to view the film with naked eye (at the position E — see Fig. 15.4) then only light rays reflected from a small position QR of the film will reach the eye. The image formed at the retina will be dark or bright depending on the value of Δ (see Eq. 15.5).

Interference by plane parallel film when illuminated by a point source (Fringes of equal inclination)

In Sec. 15.2, we had considered the incidence of a parallel beam of light on a thin film and had discussed the interference produced by the waves reflected form the upper and lower surfaces of the film. We will now consider the illumination of the film by a point source of light and, once again, in order to observe the film without obstructing the incident beam, we will use a partially reflecting plate G as shown in Fig. 15.19. However, in order to study the interference pattern we may *assume* the point source S to be right above the film (see Fig. 15.20) such that the distance SK (in Fig. 15.20) is equal to $SA + AK$ (in Fig. 15.19); KA (in Fig. 15.19) and KS (in Fig. 15.20) being normal to the film. Obviously, the waves reflected from the upper surface of the film will appear to emanate from the point S' where

$$KS' = KS \quad \textbf{(15.49)}$$

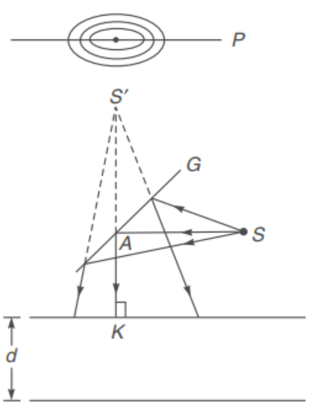


Fig. 15.19 Light emanating from a point source S is allowed to fall on a thin film of thickness d. G is a partially reflecting plate and P represents the photographic plate. On the photographic plate circular fringes are obtained.

(see Fig. 15.20). Further, simple geometrical considerations will show that the waves reflected from the lower surface will appear to emanate from the point S'', where

$$KS'' \simeq KS + 2d/n_2 \quad \textbf{(15.50)}$$

(see Fig. 15.20). Equation (15.50) is valid only for near normal incidence.* Thus, at least for near normal incidence, the interference pattern produced in region I (see Fig. 15.20) will be very nearly** the same as produced by two point coherent sources S' and S'' (which is the double hole experiment of Young discussed in the previous chapter). Thus, if we put a photographic plate P (see Fig. 15.19) we will, in general, obtain interference fringes. The intensity of an arbitrary point Q [in Fig. 15.20] will be determined by the following relations:

$$\Delta = \left(m + \frac{1}{2}\right)\lambda \qquad \text{maxima} \qquad \textbf{(15.51a)}$$

$$= m\lambda \qquad \text{minima} \qquad \textbf{(15.51b)}$$

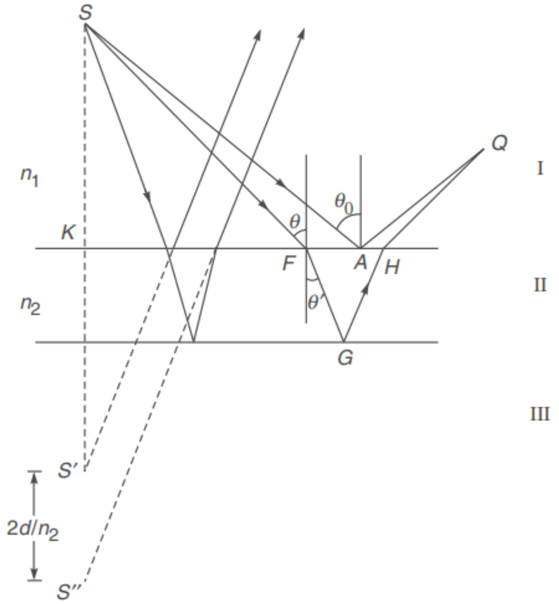


Fig. 15.20 If light emanating from a point source S is incident on a thin film then the interference pattern produced in the region I is approximately the same as would have been produced by two coherent point sources S' and S'' (separated by a distance $2d/n_2$) where d represents the thickness of the film and n_2 represents the refractive index of the film.

where $$\Delta = [n_1 SF + n_2(FG + GH) + n_1 HQ] - [n_1(SA + AQ)] \qquad \textbf{(15.52)}$$

represents the optical path difference and we have assumed that in one of the reflections, an abrupt phase change of π occurs; n_1 and n_2 are the refractive indices of media I and II respectively. The above conditions are rigorously correct; i.e., valid even for large angles of incidence. Further, it can be shown that for near normal incidence,

$$\Delta \simeq 2n_2\, d \cos\theta' \qquad \textbf{(15.53)}$$

A more rigorous calculation shows (see Ref. 15.7]

$$\Delta \simeq 2n_2\, d\cos\theta' \left[1 - \frac{n_1^2 \sin\theta \cos\theta}{n_2^2 - n_1^2 \sin^2\theta}\left(\frac{\theta_0 - \theta}{2}\right)\right] \qquad \textbf{(15.54)}$$

where the angles θ, θ_0 and θ' are defined in Fig. 15.20.

Now, if we put a photographic plate (parallel to the surface of the film (see Fig. 15.20)) we will obtain dark and bright concentric rings (see Example 14.6).*On the other hand, if we view the film with naked eye then, for a given position of the eye, we will be able to see only a very small portion of the film; e.g., with eye at the position E and the point source at S only a portion of the film around the point B will be visible [see Fig. 15.21(a)], and this point will appear to be dark or bright as the optical path difference,

$$\Delta = n_1 SQ + n_2(QA + AB) - n_1 SB$$

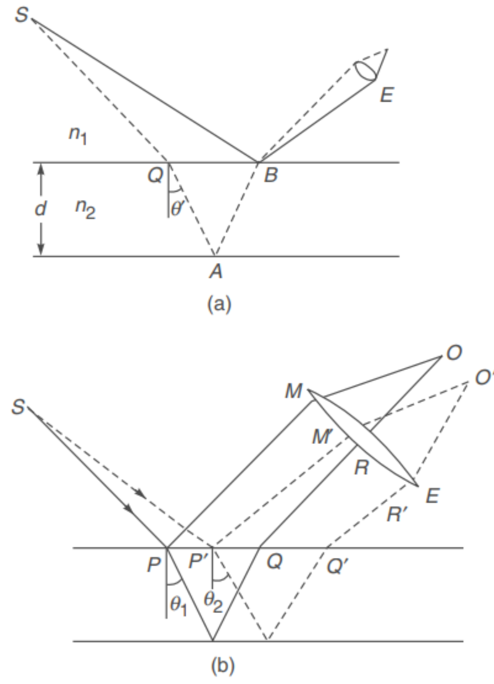


Fig. 15.21 Light emanating from a point source S is incident on a thin film; (a) if the film is viewed by the naked eye E then the point B will appear to be dark if the optical path $[\{n_1\ SQ + n_2\ (QA + AB)\} - n_1\ SB]$ is $m\lambda$, and bright if the optical path is $\left(m + \frac{1}{2}\right)\lambda$. (b) If the eye is focused for infinity then it receives parallel rays from different directions corresponding to different values of the angles of refraction θ' (and hence different values of the optical path difference).

is $m\lambda$ or $\left(m - \frac{1}{2}\right)\lambda$. Further, using a method similar to the one described in Sec. 15.3, we can obtain

$$\Delta \simeq 2n_2 d \cos\theta' \qquad \textbf{(15.55)}$$

Instead of looking at the film, if the eye is focussed at infinity, then the interference is between the rays which are derived from a single incident ray by reflection from the upper and the lower surfaces of the film [see Fig. 15.21(b)]. For example, the rays PM and QR, which focus at the point O of the retina, are derived from the single ray SP, and the rays $P'M'$ and $Q'R'$, which focus at a different point O' on the retina, are derived from the ray SP'. Since the angles of refraction θ_1' and θ_2' (for these two sets of rays) will be different, the points O and O' will, in general, not have the same intensity.

We next consider the illumination by an extended source of light S (see Fig. 15.22). Such an extended source may be produced by illuminating a ground glass plate by a sodium lamp. Each point on the extended source will produce its own interference pattern on the photographic plate P; these will be displaced with respect to one another; consequently, no definite fringe pattern will appear on the photographic plate. However, if we view the film with our eye, rays from all points

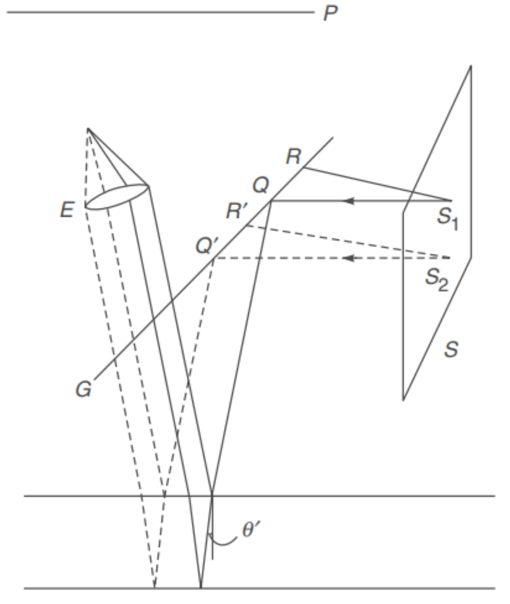


Fig. 15.22 Light emanating from an extended source illuminates a thin film. G represents the partially reflecting plate and P represents the photographic plate. The eye E is focussed at infinity.

of the film will reach the eye. If the eye is focussed at infinity then parallel light coming in a particular direction reaching the eye would have originated from nearby points of the extended source and the intensity produced on the retina would depend on the value of $2nd \cos \theta'$ which is the same for all parallel rays like S_1Q, S_2Q', etc. (see Fig. 15.22). Rays emanating in a different direction (like S_1R, S_2R', etc.) would correspond to a different value of θ' and would focus at a different point on the retina. Since θ' is constant over the circumference of a cone (whose axis is normal to the film and whose vertex is at the eye), the eye will see dark and bright concentric rings, with the center lying along the direction $\theta' = 0$. Such fringes, produced by a film of uniform thickness, are known as **Haidinger fringes**. They are also known as **fringes of equal inclination** because the changes in the optical path are due to the changes in the direction of incidence and hence in the value of θ'. In Sec. 15.10 we will discuss the Michelson interferometer where such fringes are observed.

Interference by a wedge-shaped films (Fringes of equal thickness)

Till now we have assumed the film to be of uniform thickness. We will now discuss the interference pattern produced by a film of varying thickness. Such a film may be produced by a wedge which consists of two non-parallel plane surfaces [see Fig. 15.23(a)].

We first consider a parallel beam of light incident normally on the upper surface of the film [see Fig. 15.23(a)]. In Fig. 15.23(b) the successive positions of the crests (at a particular instant of time) reflected from the upper surface and from the lower surface of the film are shown by solid and dashed lines, respectively. Obviously, a photographic plate P will record straight line interference fringes which will be parallel to the edge of the wedge (the edge is the line passing through the point O and perpendicular to the plane of the paper). The dots in the figure indicate the positions of maxima. In order to find the distance between two consecutive fringes on the film we note that for the point A to be bright*

$$n(LM + MA) = \left(m + \frac{1}{2}\right)\lambda; \quad m = 0, 1, 2, \ldots \tag{15.56}$$

[see Fig. 15.23(a)]. However, when the wedge angle ϕ is very small (which is indeed the case for practical systems)

$$LM + MA \approx 2AA'$$

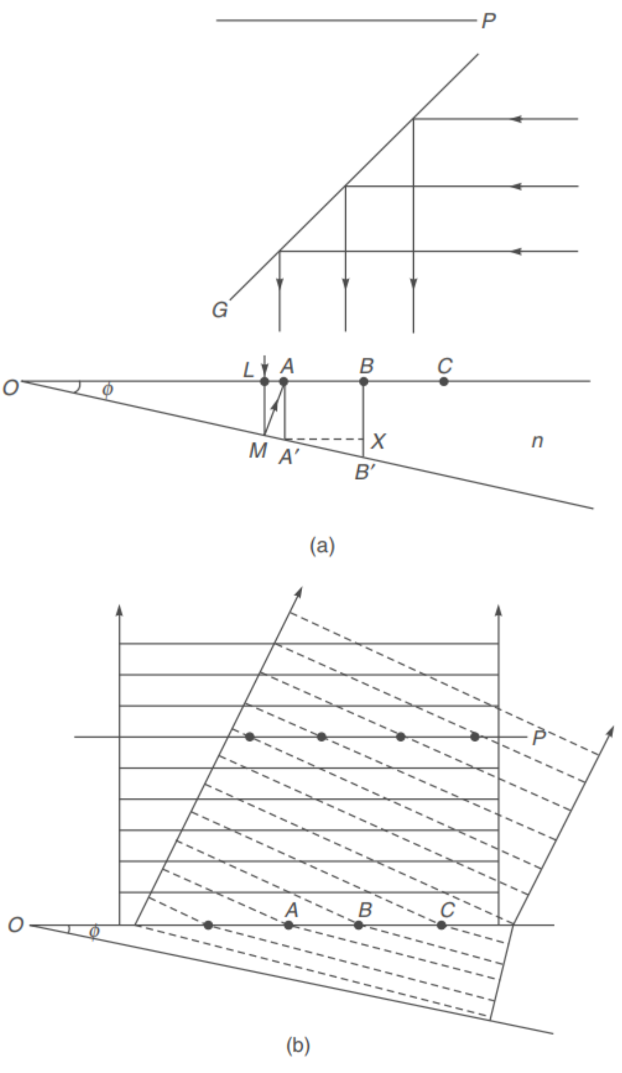


Fig. 15.23 (a) A parallel beam of light incident on a wedge. (b) The solid and the dashed lines represent the positions of the crests (at a particular instant of time) corresponding to the waves reflected from the upper surface and from the lower surface respectively. The maxima will correspond to the intersection of the solid and dashed lines. The fringes will be perpendicular to the plane of the paper.

where AA' represents the thickness of the film at A. Thus, the condition for the point A to be bright is

$$2nAA' \approx \left(m+\frac{1}{2}\right)\lambda \tag{15.57}$$

Similarly, the next bright fringe will occur at the point B where

$$2nBB' \approx \left(m+\frac{3}{2}\right)\lambda \tag{15.58}$$

Thus, $\quad 2n(BB' - AA') \approx \lambda$

or

$$XB' \approx \lambda/2n \tag{15.59}$$

But $\quad XB' = (A'X)\tan\phi$

or

$$A'X = \beta \approx \frac{\lambda}{2n\phi} \tag{15.60}$$

where β represents the fringe width and we have assumed ϕ to be small. Such fringes are commonly referred to as **fringes of equal thickness**.

On the other hand, for a point source, the fringe pattern will be similar to the parallel film case; i.e., for near normal incidence, the pattern will be very nearly the same as produced by two sources S' and S'' (Fig. 15.24). (Notice that the point S'' is not vertically below S'; this is a consequence of the fact that the two surfaces of the film are not parallel.) The intensity of an arbitrary point Q will be determined by the following equations:

$$[SA + n(AB + BC) + CQ] - [SD + DQ] = \left(m+\frac{1}{2}\right)\lambda \quad \text{maxima} \tag{15.61}$$
$$= m\lambda \quad \text{minima}$$

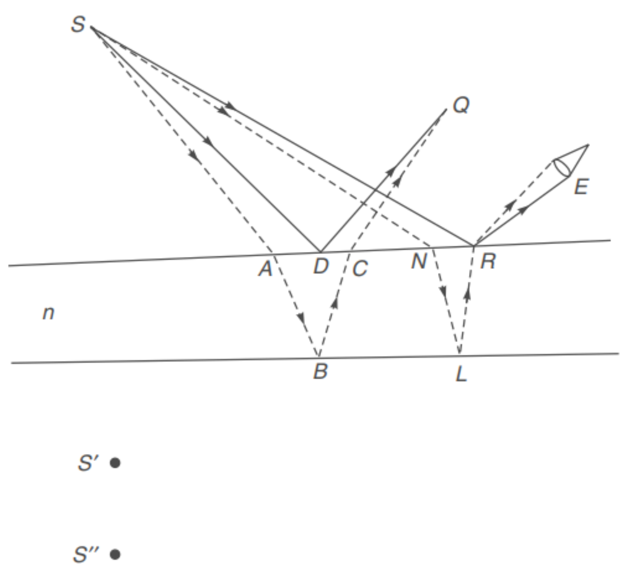


Fig. 15.24 Light from a point source illuminating a wedge. E represents the lens of the eye.

If we view the film with naked eye (say at the position E — see Fig. 15.24) then only a small portion of the film (around the point R) would be visible and the point R will be bright or dark as the optical path difference $[\{SN + n(NL + LR)\} - SR]$ is $\left(m+\frac{1}{2}\right)\lambda$ or $m\lambda$, respectively. One can similarly discuss the case when the eye is focussed for infinity.

We next consider the illumination by an extended source S as shown in Fig. 15.25. Since the extended source can be assumed to consist of a large number of independent point sources, each point source will produce its own pattern on a photographic plate P. Consequently, no definite fringe pattern will be observed.* However, if we view the film with a camera (or with a naked eye) and if the camera is focussed on the upper surface of the film then a particular point on the film will appear dark or bright depending on the fact that whether $2nd$ is $m\lambda$ or $\left(m-\frac{1}{2}\right)\lambda$ (see Fig. 15.25) — we are assuming near normal incidence. It may be seen in the figure that interference at the point Q may occur due to light coming from different points on the extended source, but if the incidence is near normal then the intensity at the point Q will be determined entirely by the thickness of the film at that

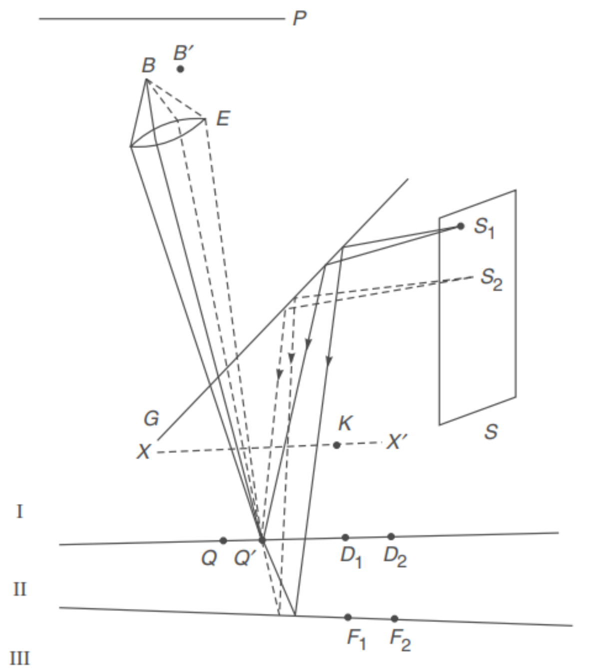


Fig. 15.25 Localized interference fringes produced by an extended source S. Fringes will be seen only when the eye is focussed on the upper surface of the film.

place. Similarly, the intensity at the point Q' will be determined by the thickness of the film at Q'; however, the point Q' will be focussed at a different point B' on the retina of the eye. The fringes will be straight lines parallel to the edge of the film OO' (Fig. 15.26). It should be emphasized that all along we are assuming near normal incidence and the fact that the wedge angle is extremely small. These assumptions are indeed valid for practical systems.

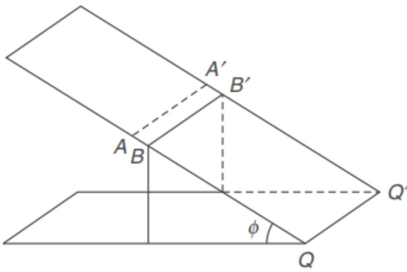


Fig. 15.26 The fringes formed by a wedge will be parallel to the edge OO'.

It is of interest to mention that if we focus the camera on a plane XX', which is slightly above the film, then no definite interference pattern will be observed. This follows from the fact that the light waves reaching the point K from S_2 undergo reflection at the points D_2 and F_2 and the light waves reaching K from S_1 undergo reflection at the points D_1 and F_1. Since the thickness of the film is not uniform, the waves reaching K from S_1 may produce brightness, whereas the waves reaching from S_2 may produce darkness. Thus, in order to view the fringes, one must focus the camera on the upper surface of the film, and in this sense, the fringes are said to be localized. It is left as an exercise for the reader to verify that if the camera is focussed for infinity, no definite interference pattern will be recorded.

Till now we have assumed the film to be 'thin'; the question now arises as to how thin the film should be. In order to obtain an interference pattern, there should be definite phase relationship between the waves reflected from the upper surface of the film and from the lower surface of the film. Thus the path difference Δ (= $2nd \cos \theta'$) should be small compared to the coherence length.* For example, if we are using the D_1 line of an ordinary sodium lamp ($\lambda = 5.890 \times 10^{-5}$ cm), the coherence length is of the order of 1 cm and for fringes to be visible Δ should be much less than 1 cm. It should be pointed out that there is no particular value of Δ for which the fringes disappear; but as the value of Δ increases, the contrast of the fringes becomes poorer. A laser beam has a very high coherence length and fringes can be visible even for path differences much greater than 1 m. On the other hand, if we use a white light source no fringes will be visible for $\Delta \gtrsim 2 \times 10^{-4}$ cm (see Sec. 14.9).

It should be pointed out that interference also occurs in region III (see Fig. 15.27) between the directly transmitted beam and the beam which comes out of the film after suffering two reflections, first from the lower surface and then from the upper surface of the film. However, the two amplitudes will be very different and the fringes will have very poor contrast (see Example 15.1).

Example 15.1 Consider a film of refractive index 1.36 in air. Assuming near normal incidence ($\theta \approx 0$), show that whereas the amplitudes of the reflected rays (1) and (5) (Fig. 15.27) are nearly equal, the amplitudes of the transmitted rays (4) and (7) are quite different. (This is the reason why the fringes observed in transmission have very poor contrast.)

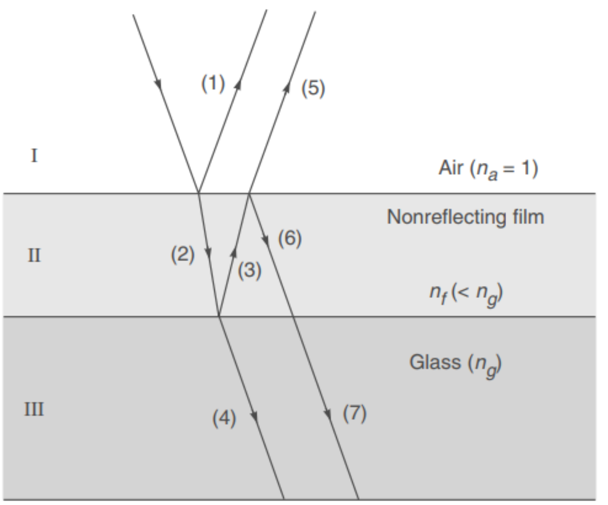


Fig. 15.27 In general, whereas the amplitude of (1) and (5) are nearly the same, the amplitudes of (2) and (6) are quite different.

Solution: Let the amplitude of the incident ray be a and let the amplitudes of the rays (1), (2), (3),…be denoted by a_1, a_2,… etc. Using Eqs. (15.10a) and (15.10b), we get

$$a_1 = \frac{1-n}{1+n}a = -\frac{0.36}{2.36}a \approx -0.153a$$

$$a_2 = \frac{2}{1+n}a = \frac{2}{2.36}a \approx 0.847a$$

$$a_3 = \frac{n-1}{n+1}a_2 = \frac{0.36}{2.36} \times 0.874a \approx 0.129a$$

$$a_5 = \frac{2n}{n+1}a_3 = \frac{2 \times 1.36}{2.36} \times 0.129a \approx 0.149a$$

$$a_4 = \frac{2n}{1+n}a_2 = \frac{2 \times 1.36}{2.36} \times 0.847a \approx 0.977a$$

$$a_7 = \frac{2n}{n+1}a_6 = \frac{2n}{n+1} \cdot \frac{n-1}{n+1}a_3 = \frac{2 \times 1.36 \times .36}{(2.36)^2}a_3$$

$$\approx 0.023a$$

We first note that the sign of a_5 is opposite to that of a_1 which is a consequence of the fact that a sudden phase change of π occurs when the ray gets reflected at the point B. Further the magnitude of a_5 is nearly equal to that of a_1. On the other hand $|a_7| << |a_4|$. This is the reason why the interference fringes formed in transmission have poor contrast.

Newton's Rings

An important application of interference in thin films is the formation of Newton's rings. When a plano convex lens of long focal length is placed over an optically plane glass plate, a thin air film with varying thickness is enclosed between them. The thickness of the air film is zero at the point of contact and gradually increases outwards from the point of contact. When the air film is illuminated by monochromatic light normally, alternate bright and dark concentric circular rings are formed with dark spot at the centre. These rings are known as Newton's rings. When viewed with white light, the fringes are coloured

The Experiment

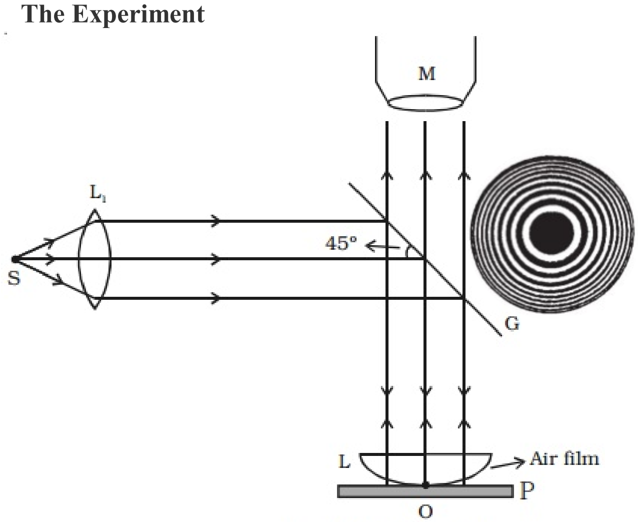


Fig. 15.28 The Newton's Ring set-up

Fig. 15.28 shows an experimental arrangement for producing and observing Newton's rings. A monochromatic source of light S is kept at the focus of a condensing lens L_1. The parallel beam of light emerging from L_1 falls on the glass plate G kept at 45°. The glass plate reflects a part of the incident light vertically downwards, normally on the thin air film, enclosed by the Plano-convex lens L and plane glass plate P. The reflected beam from the air film is viewed with a microscope. Alternate bright and dark circular rings with dark spot as centre is seen.

Theory

The formation of Newton's rings can be explained on the basis of interference between waves which are partially reflected from the top and bottom surfaces of the air film. If *t* is the thickness of the air film at a point on the film, the refracted wavelet from the lens has to travel a distance *t* into the film and after reflection from the top surface of the glass plate, has to travel the same distance back to reach the point again.

Thus, it travels a total path *2t*. One of the two reflections takes place at the surface of the denser medium and hence it introduces an additional phase change of π or an equivalent path difference λ/2 between two wavelets.

∴ The condition for brightness is,

$$\text{Path difference, } \delta = 2t + \frac{\lambda}{2} = n\lambda$$

$$\therefore \quad 2t = (2n-1)\frac{\lambda}{2}$$

where n = 1, 2, 3 ... and λ is the wavelength of light used.

The condition for darkness is,

$$\text{path difference} \quad \delta = 2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore \quad 2t = n\lambda$$

where n = 0, 1, 2, 3 ....

The thickness of the air film at the point of contact of lens L with glass plate P is zero. Hence, there is no path difference between the interfering waves. So, it should appear bright. But the wave reflected from the denser glass plate has suffered a phase change of π while the wave reflected at the spherical surface of the lens has not suffered any phase change. Hence the point O appears dark. Around the point of contact alternate bright and dark rings are formed.

Expression for the radius of the n^{th} dark ring

Let us consider the vertical section SOP of the Plano-convex lens through its centre of curvature C, as shown in Fig 5.20. Let R be the radius of curvature of the Plano-convex lens and O be the point of contact of the lens with the plane surface. Let t be the thickness of the air film at S and P. Draw ST and PQ perpendiculars to the plane surface of the glass plate. Then ST = AO = PQ = *t*

Let r_n be the radius of the n^{th} dark ring which passes through the points S and P.

Then SA = AP = r_n

If ON is the vertical diameter of the circle, then by the law of segments

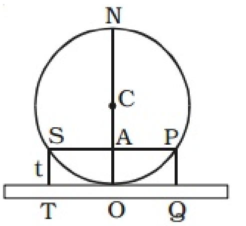


Fig. 15.29 Radius of Newton's Ring

$$SA.\ AP = OA.\ AN$$

$$r_n^2 = t(2R-t)$$

$$r_n^2 = 2Rt \text{ (neglecting } t^2 \text{ comparing with } 2R)$$

$$2t = \frac{r_n^2}{R}$$

According to the condition for darkness

$$2t = n\lambda$$

$$\therefore \quad \frac{r_n^2}{R} = n\lambda$$

$$r_n^2 = nR\lambda \quad \text{or} \quad r_n = \sqrt{nR\lambda}$$

Since R and λ are constants, we find that the radius of the dark ring is directly proportional to square root of its order. i.e. $r_1 \propto \sqrt{1}$, $r_2 \propto \sqrt{2}$, $r_3 \propto \sqrt{3}$, and so on. It is clear that the ring gets closer as n increases.

Applications of Newton's rings

(i) Using the method of Newton's rings, the wavelength of a given monochromatic source of light can be determined. The radius of n^{th} dark ring and $(n+m)^{th}$ dark ring are given by

$$r_n^2 = nR\lambda \quad \text{and} \quad r^2_{n+m} = (n+m)R\lambda$$

$$r_{n+m}^2 - r_n^2 = mR\lambda$$

$$\therefore \quad \lambda = \frac{r_{n+m}^2 - r_n^2}{mR}$$

Knowing r_{n+1}, r_n and R, the wavelength can be calculated.

(ii) Using the method of Newton's rings, the refractive index of a material can be calculated. Let λ_a and λ_n represent the wavelength of light in air and in the medium (liquid). If r_n is the radius of the n^{th} dark ring and r'_n is the radius of the n^{th} dark ring in liquid, then

$$r_n^2 = nR\lambda_a$$

$$r'^2_n = nR\lambda_m = \frac{nR\lambda_a}{\mu} \qquad \left[\because \mu = \frac{\lambda_a}{\lambda_m}\right]$$

$$\therefore \quad \mu = \frac{r_n^2}{r'^2_n}$$

Michelson Interferometer

A schematic diagram of the Michelson interferometer is shown in Fig. 15.34. S represents a light source (which may be a sodium lamp) and L represents a ground glass plate so that an extended source of almost uniform intensity is formed. G_1 is a beam splitter; i.e., a beam incident on G_1 gets partially reflected and partially transmitted. M_1 and M_2 are good quality plane mirrors having very high reflectivity. One of the mirrors (usually M_2) is fixed and the other (usually M_1) is capable of moving away or towards the glass plate G_1 along an accurately machined track by means of a screw. In the normal adjustment of the interferometer, the mirrors M_1 and M_2 are perpendicular to each other and G_1 is at 45° to the mirror.

Waves emanating from a point P get partially reflected and partially transmitted by the beam splitter G_1, and the two resulting beams are made to interfere in the following manner: The reflected wave [shown as (1) in Fig. 15.34] undergoes a further reflection at M_1 and this reflected wave gets (partially) transmitted through G_1; this is shown as (5) in the figure. The transmitted wave [shown as (2) in Fig. 15.34] gets reflected by M_2 and gets (partially) reflected by G_1 and results in the wave shown as (6) in the figure. Waves (5) and (6) interfere in a manner exactly similar to that shown in Fig. 15.22. This can easily be seen from the fact that if x_1 and x_2 are the distances of the mirrors M_1 and M_2 from the plate G_1, then to the eye the waves emanating from the point P will appear to get reflected by two parallel mirrors [M_1 and M_2' – see Fig. 15.34] separated by a distance ($x_1 \sim x_2$). As discussed in Sec. 15.7, if we use an extended source, then no definite interference pattern will be obtained on a photographic plate placed at the position of the eye. Instead, if we have a camera focused for infinity, then on the focal plane we will obtain circular fringes, each circle corresponding to a definite value of θ (see Figs. 15.22 and 15.35); the circular fringes will look like the ones shown in Fig. 15.36. Now, if the beam splitter is just a simple glass plate, the beam reflected from the mirror M_2 will undergo an abrupt phase change of π (when getting reflected by the beam splitter) and since the extra path that one of the beams will traverse will be $2(x_1 \sim x_2)$, the condition for destructive interference will be

$$2\,d\cos\theta = m\,\lambda$$

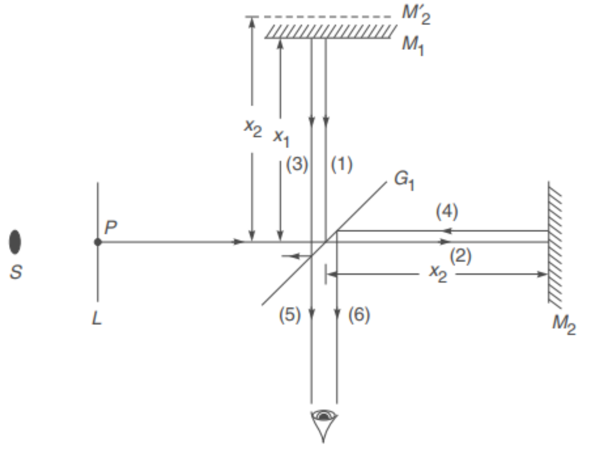


Fig. 15.34 Schematic of the Michelson interferometer.

where $m = 0, 1, 2, 3, \ldots\ldots$ and

$$d = x_1 \sim x_2$$

and the angle θ represents the angle that the rays make with the axis (which is normal to the mirrors as shown in Fig. 15.35). Similarly, the condition for a bright ring would be

$$2\,d\cos\theta = \left(m + \frac{1}{2}\right)\lambda$$

For example, for $\lambda = 6 \times 10^{-5}$ cm if $d = 0.3$ mm, the angles at which the dark rings will occur will be

$$\theta = \cos^{-1}\left(\frac{m}{1000}\right)$$

$$= 0°, 2.56°, 3.62°, 4.44°, 5.13°, 5.73°, 6.28°, \ldots$$

corresponding to m = 1000, 999, 998, 997, 996, 995, … Thus the central dark ring in Fig. 15.36(a) corresponds to $m = 1000$, the first dark ring corresponds to $m = 999$, etc. If we now reduce the separation between the two mirrors so that $d = 0.15$ mm, the angles at which the dark rings will occur will be [see Fig.15.36(b)]

$$\theta = \cos^{-1}\left(\frac{m}{500}\right) = 0°, 3.62°, 5.13°, 6.28°, 7.25°, \ldots$$

where the angles now correspond to m = 500, 499, 498, 497, 496, 495, …Thus as we start reducing the value of d, the fringes will appear to collapse at the centre and the fringes become less closely placed. It may be noted that if d is now slightly decreased, say from 0.15 mm to 0.14985 mm,

$$2\,d = 499.5\,\lambda$$

the dark central spot in Fig.15.36(b) (corresponding to m = 500) would disappear and the central fringe will become bright. Thus, as d decreases, the fringe pattern tends to collapse towards the centre. (Conversely, if d is increased, the fringe pattern will expand.)

Determination of wavelength:

If N fringes collapse to the center as the mirror M_1 moves by a distance d_o, then we must have

$$2\,d = m\,\lambda$$
$$2\,(d - d_0) = (m - N)\,\lambda$$

where we have put $\theta' = 0$ because we are looking at the central fringe. Thus,

$$\lambda = \frac{2d_0}{N} \qquad \textbf{(15.73)}$$

This provides us with a method for the measurement of the wavelength. For example, in a typical experiment, if one finds 1000 fringes collapse to the center as the mirror is moved through a distance of 2.90×10^{-2} cm, then

$$\lambda = 5800\ \text{Å}$$

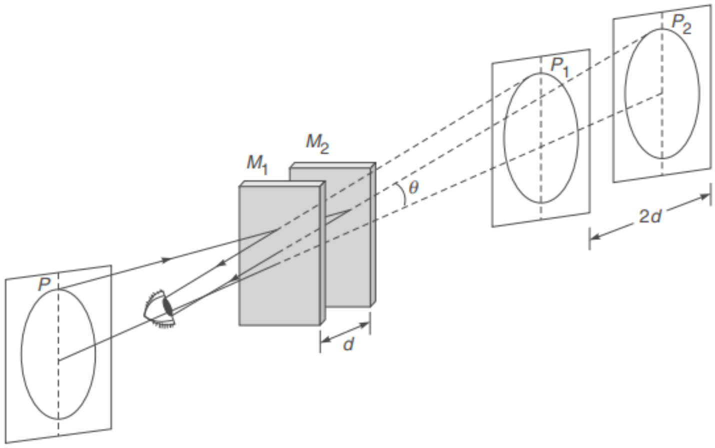


Fig. 15.35 A schematic of the formation of circular fringes [Adapted from Ref. 15.7].

The above method was used by Michelson for the standardization of the meter. He had found that the red cadmium line (λ = 6438.4696 Å) is one of the ideal monochromatic sources and as such this wavelength was used as a reference for the standardization of the meter. In fact, he defined the meter by the following relation:

1 meter = 1553164.13 red cadmium wavelengths,

the accuracy is almost one part in 10^9.

In an actual Michelson interferometer, the beam splitter G_1 consists of a plate (which may be about 1/2 cm thick), the back surface of which is partially silvered and the reflections occur at the back surface as shown in Fig. 15.37. It is immediately obvious that the beam (5) traverses the glass plate thrice and in order to compensate for this additional path, one introduces a 'compensating plate' G_2 which is exactly of the same thickness as G_1. The compensating plate is not really necessary for a monochromatic source because the additional path $2(n - 1)t$ introduced by G_1 can be compensated by moving the mirror M_1 by a distance $(n - 1)t$ where n is the refractive index of the material of the glass plate G_1.

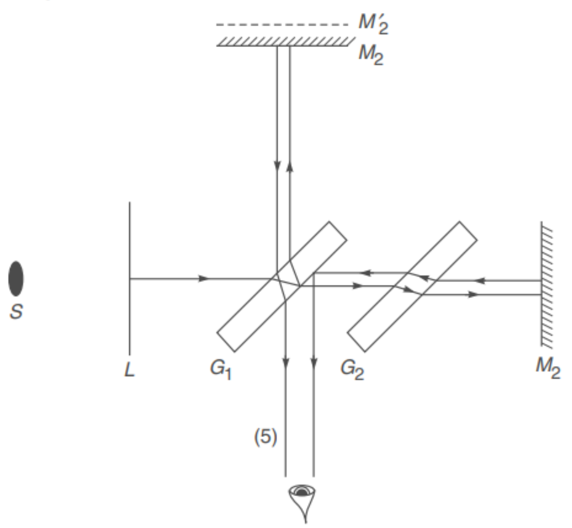


Fig. 15.37 In an actual interferometer there is also a compensating plate G_2.

However, for a white light source it is not possible to simultaneously satisfy the zero path-difference condition for all wavelengths, since the refractive index depends on wavelength. For example, for λ = 6560 Å and 4861 Å, the refractive index of crown glass is 1.5244 and 1.5330 respectively. If we are using a 0.5 cm thick crown glass plate as G_1, then M_1 should be moved by 0.2622 cm for λ = 6560 Å and by 0.2665 cm for λ = 4861 Å, the difference between the two positions corresponding to over hundred wavelengths! Thus, if we have a continuous range of wavelengths from 4861 Å to 6560 Å, the path difference between any pair of interfering rays (see Fig. 15.34) will vary so rapidly with wavelength that we would observe only a uniform white light illumination. However, in the presence of the compensating plate G_2, one would observe a few colored fringes around the point corresponding to zero path difference (see Sec. 14.9).

Determination of wavelength difference:

Michelson interferometer can also be used in the measurement of two closely spaced wavelengths. Let us assume that we have a sodium lamp which emits predominantly two closely spaced wavelengths 5890 Å and 5896 Å. The interferometer is first set corresponding to the zero path difference.* Near $d = 0$, both the fringe patterns will overlap. If the mirror M_1 is moved away (or towards) the plate G_1 through a distance d, then the maxima corresponding to the wavelength λ_1 will not, in general, occur at the same angle as λ_2. Indeed, if the distance d is such that

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = \frac{1}{2} \qquad \textbf{(15.74)}$$

and if $2d \cos \theta' = m\lambda_1$, then $2d \cos \theta' = \left(m + \frac{1}{2}\right)\lambda_2$. Thus, the maxima of λ_1 will fall on the minima of λ_2 and conversely, and the fringe system will disappear. It can easily be seen that if

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1 \qquad \textbf{(15.75)}$$

then interference pattern will again reappear. In general, if

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is 1/2, 3/2, 5/2,…we will have disappearance of the fringe pattern and if it is equal to 1, 2, 3,…then the interference pattern will appear.

Instead of two discrete wavelengths, if the source consists of all wavelengths, lying between λ and $\lambda + \Delta\lambda$, then no interference pattern will be observed if

$$\frac{2d}{\lambda} - \frac{2d}{\lambda + \frac{\Delta\lambda}{2}} \gtrsim \frac{1}{2}$$

or

$$2d \gtrsim \frac{\lambda^2}{\Delta\lambda} \quad \textbf{(15.76)}$$

In this case the fringes will not reappear because we have a continuous range of wavelengths rather than two discrete wavelengths (see Sec. 17.2).

Example 15.4 For a sodium lamp, the distance traversed by the mirror between two successive disappearances is 0.289 mm. Calculate the difference in the wavelengths of the D_1 and the D_2 lines. Assume λ = 5890 Å.

Solution: When the mirror moves through a distance 0.289 mm, the additional path introduced is 0.578 mm. Thus,

$$\frac{0.578}{\lambda} - \frac{0.578}{\lambda + \Delta\lambda} = 1$$

or

$$\Delta\lambda \simeq \frac{\lambda^2}{0.578} = \frac{(5890 \times 10^{-7})^2}{0.578} \text{ mm}$$
$$\simeq 6 \text{ Å}$$

Determination of refractive index of glass plate:

Consider a thin glass plate of thickness t and refractive index n, inserted normal to the path of one of the two interfering beams in Michelson interferometer. The optical path length of the beam through the plate is nt, while the optical path length through an equal thickness of air is just t, so the increase in optical path length caused by inserting the plate is (n–1)t. The beam traverses the plate twice, so the total path difference will be 2(n–1)t. If N is the number of fringes displaced by inserting the plate, then Nλ = 2(n–1)t.

After adjusting the mirrors to obtain circular fringes with a single central dark spot, the plate is introduced into the path of one of the interfering beams and the fringes are displaced. M1 is moved a distance d closer, until a single central dark spot is again obtained. The distance d moved is noted and the number of fringes N that disappear is counted. Then, since the insertion of the glass plate increased the optical path length by 2(n–1)t, and the mirror motion decreased it by 2d, 2d must equal 2(n–1)t, so the refractive index n of the plate can be calculated from Nλ = 2d = 2(n–1)t.

With a laser, the light source is more precisely monochromatic, so the measurement of n can be more accurate. In the Michelson interferometer, if N fringes are displaced when the plate is rotated through an angle θ from its original orientation normal to the path, the refractive index of the plate is

$$n = \frac{(2t - N\lambda)(1 - Cos\,\theta) + \frac{N^2\lambda^2}{4t}}{2t(1 - Cos\,\theta) - N\lambda} \quad (3)$$

where t is the thickness of the plate and λ is the wavelength of the laser. The last term in the numerator is often neglected. However, this causes an error that increases with θ, reaching ~1% at θ = 30°, potentially destroying the increased accuracy gained by using a laser light source.

The above equation can be approximated to get

$$n = \frac{(2t - N\lambda)(1 - Cos\,\theta)}{2t(1 - Cos\,\theta) - N\lambda} \quad \textbf{(4)}$$

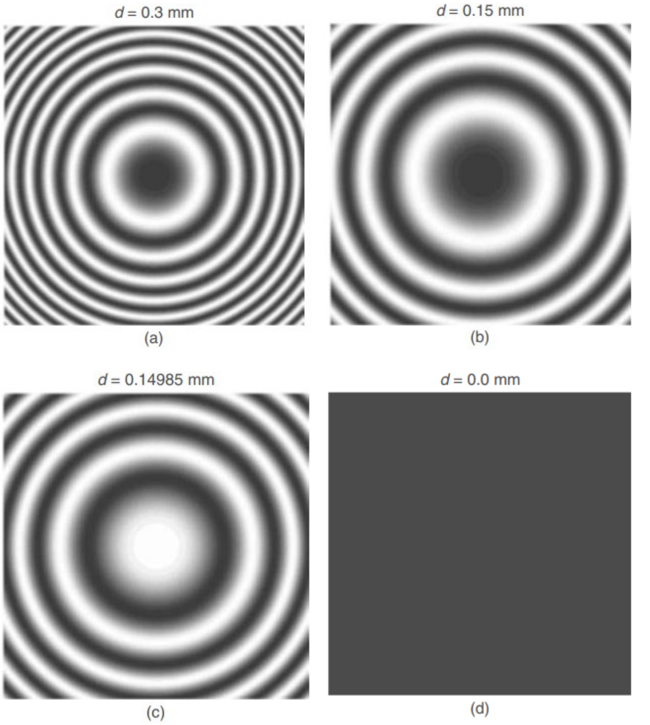


Fig. 15.36 Computer generated interference pattern produced by a Michelson interferometer.

References:

1. Optics, Ajoy Ghatak, 2017, Tata McGraw Hill
2. https://www.brainkart.com/article/Newton-s-rings---Experiment,-Theory_566/