Solution of Maxwell's eqn in free space :-

OR

Electromagnetic Waves in Vaccuum :-

Maxwell's eqn in free space,

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{———} (1)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{———} (2)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{———} (3)$$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial E}{\partial t} \quad \text{———} (4)$$

Maxwell's eqn are four first order differential eqn.
We need to convert these first " " " into 2
IInd order diff. eqn.

Take the curl of eqn (iii),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

Put the value of $\vec{\nabla}\times\vec{B}$ from eqn (4) into this eqn,

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$\nabla\cdot E = 0$ from eqn (1),

$$\nabla^2 E = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} = 0} \quad \text{———} (5)$$

Now, Take the curl of eqn (4),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \qquad (\text{from}(3))$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = -\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2} \qquad (\nabla\cdot B) = 0$$

$$\boxed{\nabla^2 B - \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2} = 0} \quad \text{———} (6)$$

eqn (5) & (6) are the IInd order diff. eqn of electric & mag. field.

All the information of 4 Maxwell eqⁿ contained in two eqⁿs (5 & 6)
eqⁿ (5) & (6) are the Wave eqⁿs of Electric & mag. field.
General Wave eqⁿ,

$$\boxed{\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0} \quad \text{——— (7)}$$

Compare (5) & (6) with (7) ⟹ We get

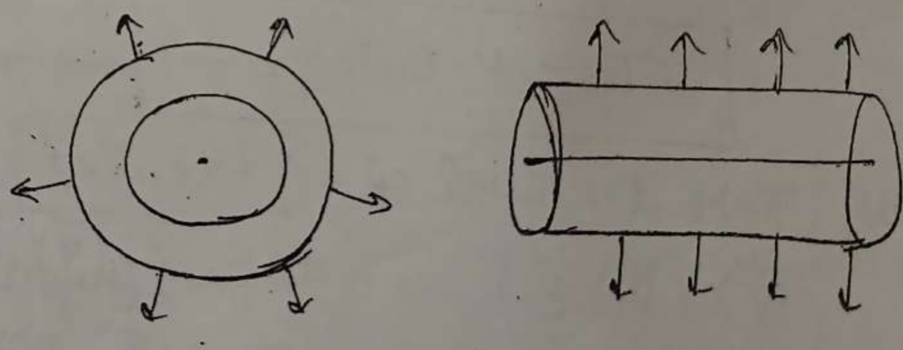
$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = C} \quad \text{——— (8)}$$

C → speed of light.

Hence, Light Waves are Electromagnetic Waves.
They don't require any medium to propagate.
[This can't be proved if Maxwell did'nt give Displacement current term]

- Solution of II nd order diff. eqⁿ gives → one give Ele. field & another give mag. field. And both field depend on each other. They can not exist without each other.

Solution may be of many kind.
In Cartisian co-ordinate → we get plane waves
" sperical-polar " → spherical waves
" cylendrical " → cylendrical wave



Small portion of spherical wave frount ~~& cylendri~~ is a plane.
(large radius sphere)
We are interested in plane wave soln.

Plane Wave Sol^n of $\vec{E}$ & $\vec{B}$

suppose $\vec{E}$ & $\vec{B}$ are propogate in x, y, z dir^n

$$\vec{E} = E_0 \, e^{i(\vec{K}\cdot\vec{r} - \omega t)}$$

$$\vec{B} = B_0 \, e^{i(\vec{K}\cdot\vec{r} - \omega t)}$$