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MPHYCC10 – 07

Electron spin: →

Bohr-Sommerfeld quantum theory of elliptic orbits with relativity correction could explain fine structure of hydrogen spectral lines. Even it suffers from two drawback

(a): Sommerfeld's relativistic correction could not be applied to the spectral lines of atoms other than Hydrogen.

(b): Simple quantum theory cound not explain anamalous Zeeman effect.

(c) To remove these two drawback Goudsmit & Uhlenbeck (1925) proposed that an electron must be looked upon as a charged sphere spinning about its own axis having an intrinsic angular momentum & consequently an intrinsic magnetic dipole moment.

spin angular momentum $= \vec{S}$

$S = \sqrt{s(s+1)}\frac{h}{2\pi}$ where $s = \pm\frac{1}{2} \Rightarrow S = \sqrt{\frac{1}{2}(\frac{1}{2}+1)}\frac{h}{2\pi} = \frac{\sqrt{3}}{2}\frac{h}{2\pi}$ — ①

spin magnetic dipole moment $= \vec{\mu}_s$

component of $\vec{S}$ along a magnetic field ‖ to the z-direction.

$$S_z = m_s \frac{h}{2\pi} \quad \text{-- ②}$$

m_s = spin magnetic quantum number

possible values of $m_s = 2s+1 = 2\times\frac{1}{2}+1 = 2$

$$m_s = +\frac{1}{2} \quad \& \quad m_s = -\frac{1}{2}$$

$$S_z = \frac{+1}{2}\frac{h}{2\pi} \quad \& \quad S_z = -\frac{1}{2}\frac{h}{2\pi} \quad \text{-- ③}$$

Now $\frac{\mu_s}{S} = -2\frac{e}{2m} \Rightarrow \vec{\mu}_s = -2\frac{e}{2m}\vec{S}$ -- ④

$\vec{\mu}_s$ & $\vec{S}$ directed in opposite direction hence there is –ve sign in the eqn ④.

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$\vec{\mu}_s = -g_s\left(\frac{e}{2m}\right)\vec{S}$ --- (5)

$g_s = 2$ = spin 'g' factor.

$\vec{\mu}_s = -g_s \frac{\mu_B}{h/2\pi}\vec{S}$

possible component of $\vec{\mu}_s$ along z axis can be given by

$\vec{\mu}_{sz} = -g_s \frac{\mu_B}{h/2\pi} m_s \frac{h}{2\pi}$ $\quad g_s = 2$ & $m_s = \pm\frac{1}{2}$

$\mu_{sz} = -g_s \mu_B m_s$

$$\boxed{\mu_{sz} = \pm \mu_B}$$

spinning e^- could successfully explained fine structre, anomlous Zeeman effect & other atomic effects also

* Theoretical prediction of electron spin was given by Goudsmit & Uhlenback in 1925.
* Stern-Gerlack experiment gives the experimental evidence of e^- spin.

Vector model of atom :- coupling of orbital and spin angular momenta

An e^- in an atom have two types of motion — orbital motion & spin motion.

orbital angular momentum $L = \sqrt{l(l+1)}\, h/2\pi$ --- (1)

z component of L $\quad l_z = m_l \frac{h}{2\pi}$ --- (2)

possible value of $m_l = (2l+1) = l, l-1 \ldots 0, -l+1, -l$ -- (3)

similarly the magnitude of spin angular

$S = \sqrt{s(s+1)}\, h/2\pi$ -- (4)

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z component of S

$S_z = m_s \frac{h}{2\pi}$ --- (6) possible value of $m_s = (2s+1)$, $s = \pm\frac{1}{2}$

Now the total angular momentum of one e^- atom $\vec{J}$ is the vector sum of $\vec{L}$ & $\vec{S}$

$$\vec{J} = \vec{L} + \vec{S} \quad \text{--- (7)}$$

The magnitude of z component of $\vec{J}$ can be given as follows

$$J = \sqrt{J(J+1)}\,\frac{h}{2\pi} \quad \text{--- (8)}$$

J = inner quantum number

$$J_z = m_J \frac{h}{2\pi} \quad \text{--- (9)}$$

m_J = corresponding to J magnetic quantum number.

possible value of $m_J = (2J+1)$ --- (10)
$= J, J-1 \ldots 0\ -J+1, -J$

Similarly

$$J_z = L_z \pm S_z \quad \text{--- (11)}$$

$$m_J = m_l \pm m_s \quad \text{--- (12)}$$

m_l = integer, m_s = half integer

$$J = l \pm s \quad \text{--- (13)}$$

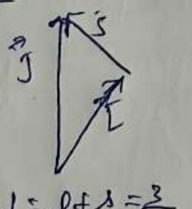
m_J = always half integer.

$\vec{J}$, $\vec{L}$ & $\vec{S}$ are always quantized only two orientation is possible here for one electron atom,

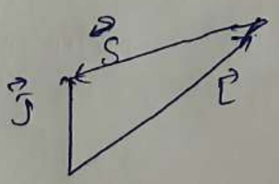
$J = l + s$ so that $J > L$

$J = l - s$ so that $J < L$

$l = 1, s = \frac{1}{2}$ $\quad j = l + s = \frac{3}{2}$, $\quad l = 1, s = -\frac{1}{2}$ $\quad J = 1 - \frac{1}{2} = \frac{1}{2}$



$J = l + s = \frac{3}{2}$



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Spin-orbit Interaction

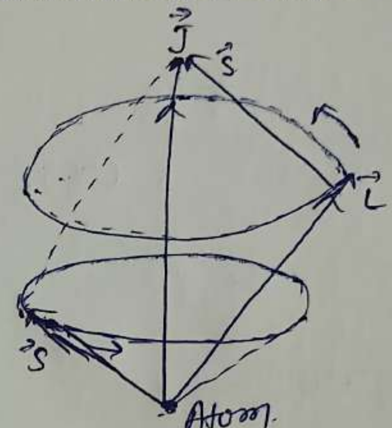
Angular momenta of the atomic electron $\vec{l}$ & $\vec{s}$ interact magnetically known as spin-orbit interaction. They excerts torque on each other which will not change the magnitude of $\vec{l}$ & $\vec{s}$ but cause them to precess around $\vec{J}$. In absence of external torque magnitude and direction of $\vec{J}$ remains constant. Angle between $\vec{l}$ & $\vec{s}$ remains invariant

$$\vec{J} = \vec{l} + \vec{s}$$

$$J^2 = l^2 + s^2 + 2ls\cos(\vec{l}, \vec{s})$$

$$\therefore \cos(\vec{l}\ \vec{s}) = \frac{J^2 - l^2 - s^2}{2ls}$$

$$= \frac{J(J+1) - l(l+1) - s(s+1)}{2\sqrt{l(l+1)}\sqrt{s(s+1)}}$$



This is the vector atom model of one-e^- atom which can be extended to many e^- atom.

The vector atom model enables us to explain fine structure of spectral lines, anomalous Zeeman effect & hyperfine structure.

Precession of $\vec{J}$ around the magnetic field:-

In the vector atom model, $\vec{l}$ & $\vec{s}$ precess around $\vec{J}$. When the atom is placed in an external magnetic field $\vec{B}$ then $\vec{J}$ precess around $\vec{B}$. While $\vec{l}$ & $\vec{s}$ precess around $\vec{J}$. The discrete orientation of $\vec{J}$ w.r.t $\vec{B}$ [w.r.t → with respect to] involves slightly different energy give rise to anomalous Zeeman effect.

Hyperfine structure of spectral lines:→

Similar to e^- the nuclei of atom have also smaller intrinsic spin angular momenta & magnetic moments. Addition of these to atomic model, experimentally observed hyperfine structure can be explained.

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precession of $\vec{l}$ & $\vec{s}$ around $\vec{J}$ & precession of $\vec{J}$ around $\vec{B}$ is shown in the following fig.