Energy Flux or Poynting Vector ($\vec{S}$) :-

Poynting Vector is defined as Energy per unit area per unit time carried by the electromagnetic wave.

OR Power per unit area is called Poynting Vector.

Unit :- J/m^2-sec or W/m^2 (watt/m2)

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \frac{EB}{\mu_0} \hat{z}$$

dir^n of $E \rightarrow \hat{x}$
$B \rightarrow \hat{y}$
$(\hat{x} \times \hat{y} \rightarrow \hat{z})$

$$B = \frac{E}{C} \Rightarrow \vec{S} = \frac{E^2}{\mu_0 C} \hat{z}$$

$$\vec{S} = \frac{E^2}{\mu_0 C} \times \frac{C}{C} \hat{z} = \frac{E^2 C}{\mu_0 C^2} = \frac{E^2 \cancel{\mu_0} \epsilon_0 C}{\cancel{\mu_0}}$$

$$\vec{S} = C \epsilon_0 E^2 \hat{z}$$

$$\boxed{\vec{S} = cu\hat{z}} \qquad (u = \epsilon_0 E^2)$$

This is Relation b/w Poynting vector & energy density.

$\vec{S} \rightarrow$ tells the dir^n of Energy propogation.

$\vec{K} \rightarrow$ " " " wave "

Generally, dir^n of $\vec{S}$ & $\vec{K}$ matches but not every time.

In present case, dir^n of $\vec{K} = \hat{z}$
dir^n of $\vec{S} = \hat{z}$

In free space, dir^n of wave propogation is same as dir^n of energy flow.

Electromag. wave not only carry the energy but also carry the momentum.

Energy Density :- (u) Energy per unit volume.

Electric field vector gives the Electric energy density u_e
& magnetic " " " " magnetic " " u_m.

$$u_e = \frac{\varepsilon_0}{2}\int_{all\ space} E^2 d\tau \qquad E \rightarrow \text{Total elec. field}$$

$$\boxed{u_e = \frac{\varepsilon_0}{2} E^2}$$

$$\& \quad u_m = \frac{1}{2\mu_0}\int_{all\ space} B^2 d\tau$$

$$\boxed{u_m = \frac{B^2}{2\mu_0}}$$

We have, $\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$ $\qquad \vec{K} \rightarrow K\hat{z}$, $\vec{E} \rightarrow E\hat{x}$ $\qquad \{\hat{z}\times\hat{x} = \hat{y}\}$.

$$\vec{B} = \frac{KE}{\omega}\hat{y}$$

$\boxed{v = \frac{\omega}{K}}$ = wave velocity (in free space)

$$\text{so} \quad \vec{B} = \frac{E}{c}\hat{y}$$

$$\therefore \quad u_m = \frac{B^2}{2\mu_0} = \frac{E^2}{2c^2\mu_0} = \frac{E^2\mu_0\varepsilon_0}{2\mu_0} = \frac{\varepsilon_0 E^2}{2}$$

$$\boxed{u_m = u_e}$$

Mag. field energy = Electric field energy.
In free space, Mag. field & Ele. field carry equal energy. This energy remain in field, i.e. mag. energy remain in mag. field & E-energy remain in Elec field.

Total Energy density $\boxed{u = u_e + u_m = \varepsilon_0 E^2}$

Momentum Density of EM Wave :-

Total mom. per unit volume called Mom. density. denoted by $\vec{p}$

In free Space, $\vec{p} = \mu_0 \varepsilon_0 \vec{S}$ $\quad (\vec{S} = cu)$

$$\boxed{\vec{p} = \frac{\vec{S}}{c^2} = \frac{u}{c}\hat{z}}$$

This is called Electromag. mom. density.

Average Value of these quantities :-

We have, Energy density $u = \varepsilon_0 E^2$

Poynting vector, $\vec{S} = c\,\varepsilon_0 E^2 \hat{z}$

Momentum density $p = \frac{\varepsilon_0 E^2}{c}\hat{z}$

Now, we calculate average energy density, Average poynting vector & average mom. density over a cycle.

We know, the solutions are

$$\vec{E} = E_0 e^{i(kz-\omega t)}\hat{x}$$

$$\vec{B} = B_0 e^{i(kz-\omega t)}\hat{y}$$

We can write,

$$\vec{E} = E_0[\underbrace{\cos(kz-\omega t)}_{\text{real Part}} + i\underbrace{\sin(kz-\omega t)}_{\text{imaginary Part}}]\hat{x}$$

Only Real part carries the energy.

$$E^2 = \vec{E}\cdot\vec{E} = E_0^2\cos^2(kz-\omega t)$$

Average Value of energy density

$$\langle u \rangle = \varepsilon_0 E_0^2 \langle \cos^2(kz-\omega t)\rangle$$

Average value of $\cos^2$ over a cycle of 2π gives $\frac{1}{2}$.

So $\boxed{\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2}$ → we can also write it in terms of mag. field (B_0) → amp of mag. field

E_0 → amplitude

This is the combination of electric & magnetic energy density

So seperately,

$$\langle u_e \rangle = \frac{1}{4} \varepsilon_0 E_0^2$$

$$\langle u_m \rangle = \frac{1}{4} \varepsilon_0 E_0^2$$

Relation b/w amplitude of elec. & mag. field, $B_0 = \frac{E_0}{c}$

$$\Rightarrow E_0 = B_0 c$$

$$\Rightarrow E_0^2 = B_0^2 c^2$$

$$\Rightarrow E_0^2 = \frac{B_0^2}{\mu_0 \varepsilon_0}$$

So $\boxed{\langle u \rangle = \frac{1}{\cancel{2}4} \varepsilon_0 E_0^2 = \frac{B_0^2}{4 \mu_0}}$

<u>Average Value of poynting vector</u> also called <u>Intensity</u> it is the average energy per unit area per unit time associated with the EM wave.

$$\langle \vec{S} \rangle = I$$

e.g. An EM wave of given intensity, incident on $1\ m^2$ area for 10 sec then calculate energy transfered to the surface.

$$\underbrace{I}_{\text{energy / unit area / unit time}} = 2\ J/m^2\text{-sec}$$

so Energy $= 2 \times 10 = 20$ Joule

<u>Now</u>,

$$\langle \vec{S} \rangle \equiv I \text{ (intensity)}$$

$$= c \langle u \rangle \hat{z}$$

$\boxed{\langle \vec{S} \rangle = \frac{1}{2} \varepsilon_0 c E_0^2 \hat{z}}$ → This much intensity is transfe[r]d to the surface.

This can be in another form (in terms of mag. field)

<u>Average Value of Mom. density</u>

Mom. density → mom./unit volume.

If Mom. density of EM wave is given ~~for~~ a unit volume & calculate mom. transfer to the given volume then

$$\langle \vec{p} \rangle = \frac{1}{2}\frac{1}{c}\, \varepsilon_0 \langle E_0^2 \rangle$$

$$\boxed{\langle \vec{p} \rangle = \frac{\varepsilon_0}{2} \frac{E_0^2}{c}}$$

This much avg. mom./unit volume will transfer to the volume.