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MJCPHY06 – 06

Date : 08/02/2025
Time: 09 am

- Poynting's theorem
- Definition
- Mathematical form (Differential & Integral)
- continuity equation analog
- Derivation

Poynting's theorem was developed by British physicist John Henary Poynting. It is law of conservation energy in electromagnetic field. It is strictly ~~to~~ true for non dispersive media. This theorem is analogous to the work-energy theorem in classical mechanics and mathematically similar to continuity equation.

Poynting's theorem :→ This theorem states that the rate of energy transfer per unit volume from a region of space is equal to rate of work done on the charge distribution in the region plus the energy flux leaving that region.

Mathematically

$$-\frac{\partial u}{\partial t} = \nabla\cdot S + J\cdot E \quad \text{---} (1)$$

where $-\frac{\partial u}{\partial t}$ = rate of change of energy density in a given volume.

$\nabla\cdot S$ = energy flowing out of the volume.

$J\cdot E$ = rate at which the fields do work of charges in the volume

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J = current density corresponding to the motion of charges, E = electric field.

Integral form of Poynting's theorem :→

$$\boxed{-\frac{d}{dt}\int_V u\,dV = \oiint_{\partial V} S\cdot dA + \int_V J\cdot E\,dV}$$

S = energy flow, u = energy density
∂V = boundary volume.

Continuity equ analogous :→

$$\nabla\cdot S + \varepsilon_0 E\frac{\partial E}{\partial t} + \frac{B}{\mu_0}\frac{\partial B}{\partial t} + J\cdot E = 0$$

- ε_0 = permittivity of the free space.
- μ_0 = permeability of the free space.
- $\varepsilon_0 E\cdot\frac{\partial E}{\partial t}$ - Density of reactive power driving the build up of electric field.
- $\frac{B}{\mu_0}\cdot\frac{\partial B}{\partial t}$ = Density of reactive power driving the build up of magnetic field.
- $J\cdot E$ = Density of electric power dissipated by the Lorentz force acting on charge carriers.

Derivation :→ magnitude of the infinitesimal charge $dq = \rho d^3x$ --- (1)
work done by the electromagnetic field is given by Lorentz force law

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$$dP = dF\cdot v = (E + u\times B)\,dq\cdot v = E\cdot \rho v\, d^3x + 0 = E\cdot J\, d^3x \quad \text{--- (2)}$$

because $(v\times B)\cdot v = 0$
& the volume charge density ~~D~~ ρ
& current charge density $J = \rho v$
& v = velocity of charge dq.

The rate of work done on the whole charges in the volume V is P

$$P = \int_V dP = \int_V J\cdot E\, d^3x \quad \text{--- (3)}$$

By Ampere's circuital law

$$J = \nabla\times H - \frac{\partial D}{\partial t} \quad \text{--- (4)}$$

$$\int_V J\cdot E\, d^3x = \int_V \left[E\cdot(\nabla\times H) - E\cdot\frac{\partial D}{\partial t}\right] d^3x \quad \text{- (5)}$$

Now using vector identity
$\nabla\cdot(E\times H) = (\nabla\times E)\cdot H - E(\nabla\cdot H)$ in eqn (5)

$$\int_J J\cdot E\, d^3x = -\int_V \left[\nabla\cdot(E\times H) - H\cdot(\nabla\times E) + E\cdot\frac{\partial D}{\partial t}\right] d^3x \quad \text{- (6)}$$

we know Faraday's law $\nabla\times E = -\frac{\partial B}{\partial t}$ using in (6)

$$\int_V J\cdot E\, d^3x = -\int_V \left[\nabla\cdot(E\times H) + E\frac{\partial D}{\partial t} + H\frac{\partial B}{\partial t}\right] d^3x \quad \text{--- (7)}$$

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The following assumption is also required
- charges are moving in non dispersive medium
- The total electromagnetic energy density for time varying field is given as

$$u = \frac{1}{2}\left[E\cdot D + B\cdot H\right]$$

- It can be shown that

$$\frac{\partial}{\partial t}(E\cdot D) = 2E\cdot\frac{\partial D}{\partial t}$$

$$\& \quad \frac{\partial}{\partial t}(H\cdot B) = 2H\cdot\frac{\partial B}{\partial t}$$

using all the above relation in eqn (7) we have

$$\int_V J\cdot E\, d^3x = -\int_V\left[\frac{\partial u}{\partial t} + \nabla\cdot(E\times H)\right]d^3x$$

$$\Rightarrow \boxed{-\frac{\partial u}{\partial t} = \nabla\cdot S + J\cdot E} \quad \text{as } S = E\times H$$

$S = E\times H$ is known as Poynting's vector.