Dr. Ranjana Singh
Assistant Professor Physics
Har Prasad Das Jain College
Ara, Bihar, India

Date: 07/02/2025
Time: 03 P.m

MPHYCC-10

Good afternoon everyone. Now the prerequisites needed to understand the vector atom model is completed.

Today's talk is based on the following topics

(a) Orbital magnetic dipole moment.
(b) Bohr Magneton
(c) Behaviour of magnetic dipole in external magnetic field
(d) Larmor precession
(e) space quantization
(f) Electron spin
(g) vector atom model of an atom: Coupling of orbital and spin angular momenta.

(a) Orbital magnetic dipole moment

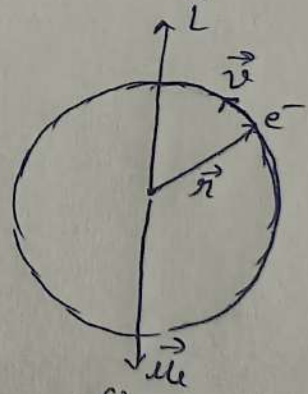

fig. 1

Let us consider one electron atom as shown in the fig. 1

l = orbital quantum number = gives the magnitude of electron's angular momentum

e^- revolving around the nucleus behave as minute current loop and produces magnetic field ie it act as a magnetic dipole. Therefore we will calculate its magnetic moment.

e^- of mass m, charge $-e$, moving with velocity 'v' in a circular Bohr orbit of radius r as shown in fig. 1.

current produced by e^-, $i = \dfrac{e}{T}$ —(1) where T = orbital period of e^-

Dr. Ranjana Singh
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$T = \frac{2\pi r}{v}$ ---- (ii), $i = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$ ---(iii)

From electromagnetic theory, magnitude of the orbital magnetic dipole moment $\vec{\mu}_l$ for a current 'i' in a loop of area A.

$$\mu_l = iA \text{ -- (iv)} \Rightarrow \mu_l = \frac{ev}{2\pi r} \cdot A \text{ -- (v)}$$

$A = \pi r^2$ -(vi) using in eqn (v)

$$\mu_l = \frac{ev}{2\cancel{\pi} \cancel{r}} \cdot \cancel{\pi} r^{\cancel{2}} = \frac{evr}{2} \quad \text{--- (vii)}$$

e^- is -vely charged particle hence $\vec{\mu}_l$ directed in the opposite direction of $\vec{L}$. magnitude of L can be given as

$$L = mvr \text{ -- (viii)}$$

Dividing eqn (vii) by (viii)

$$\boxed{\frac{\mu_l}{L} = \frac{evr}{2 \cdot mvr} = \frac{e}{2m}}$$

$$\boxed{\frac{\mu_l}{L} = \frac{e}{2m}} \text{ -(ix)}$$

From this relation it is clear that ratio of magnitude of orbital magnetic dipole to the magnitude of L of the orbital angular momentum of the e^- is constant.

$\frac{\mu_l}{L} =$ constant $= \frac{e}{2m} \Rightarrow \frac{\mu_l}{L}$ is independent of the details of the orbit.

Dr. Ranjana Singh
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$$\boxed{\frac{\mu_l}{L} = \frac{e}{2m}} \text{ —(ix) Constant = gyromagnetic ratio for the electron}$$

The above relation can be written the vector form as follows

$$\frac{\vec{\mu}_l}{\vec{L}} = -\frac{e}{2m} \Rightarrow \vec{\mu}_l = -\left(\frac{e}{2m}\right)\vec{L} \quad (x)$$

{ -ve indicate that direction of $\vec{\mu}_l$ is opposite to $\vec{L}$

Unit of $\vec{\mu}_l$ ⟶ ampere meter^2 or Joule/Tesla
(Am^2) (JT^{-1})

$$\vec{\mu}_l = -\left(\frac{e}{2m}\right)\vec{L} = -g_l\left(\frac{e}{2m}\right)\vec{L} \Rightarrow g_l = 1 = \text{orbital 'g' factor}$$

* value of orbital 'g' factor for orbital motion of electron is 1.

Bohr Magneton :-

From quantum mechanics the permitted scalar value of orbital angular momentum $\vec{L}$ can be given as

$$L = \sqrt{l(l+1)}\,\frac{h}{2\pi}$$

l = orbital quantum number
h = Planck's constant
$h = 6.63 \times 10^{-34}$ J.S

From eqn (ix) $\frac{\mu_l}{L} = \frac{e}{2m}$

$$\mu_l = \frac{e}{2m}L = \frac{e}{2m}\sqrt{l(l+1)}\,\frac{h}{2\pi}$$

$$\boxed{\mu_l = \frac{eh}{4\pi m}\sqrt{l(l+1)}} \quad (xi)$$

Dr. Ranjana Singh (Asst. Prof. physics)
HarPrasad Das Jain College
Ara, Bihar India

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The quantity $\frac{eh}{4\pi m}$ forms a natural unit for the measurement of atomic magnetic dipole moment. It is termed as Bohr magneton and denoted as μ_B

$$\mu_B = \frac{eh}{4\pi m} = \frac{1.6\times10^{-19}(C)\times 6.63\times10^{-34}(J.S)}{4\times3.14\times 9.1\times10^{-31}(kg)} = 9.27\times10^{-24}Am^2$$

$$\boxed{\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}Am^2}$$

$$\boxed{\mu_l = \sqrt{l(l+1)}\,\mu_B}$$

$\vec{\mu}_l = -g_l\left(\frac{e}{2m}\right)\vec{L}$ can be written as follows

$$\boxed{\vec{\mu}_l = -g_l\left(\frac{2\pi\mu_B}{h}\right)\vec{L}}$$

Behaviour of magnetic dipole in external magnetic field

Larmor precession :→

an e^- revolving around the nucleus of an atom is equivalent to magnetic dipole. When the atom is placed in external magnetic field. the e^- orbit precesses about the field direction. This precession is called Larmor precession and frequency of the precession is called Larmor frequency.

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Har Prasad Das Jain College
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Behaviour of magnetic dipole in external magnetic field.

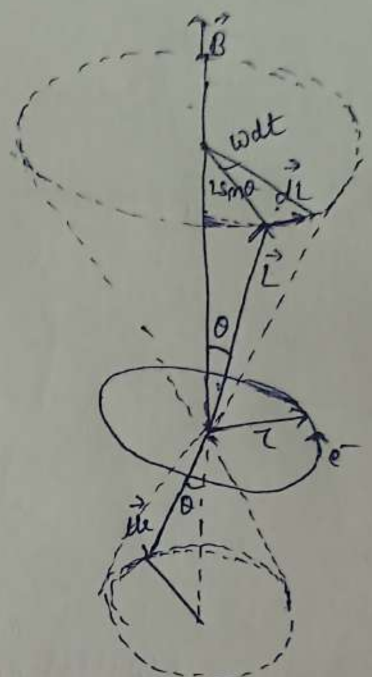

fig.2

e^- orbit is shown in external magnetic field $\vec{B}$ (fig. 2). The orbital angular momentum $\vec{L}$ is $\perp^r$ to the plane of orbit. θ is angle between $\vec{B}$ & $\vec{L}$

~~The value of orbital~~
Orbital magnetic dipole moment μ_e of e^- can be given as

$$\vec{\mu}_e = -\left(\frac{e}{2m}\right)\vec{L} \quad --- (1)$$

$e =$ charge on e^-, $m =$ mass of e^-
-ve sign indicates μ_e directed opposite to $\vec{L}$

Interaction between $\vec{L}$ & $\vec{B}$, the dipole experience torque $\vec{\tau}$

$$\vec{\tau} = \vec{\mu}_e \times \vec{B} \quad --- (2)$$

From eqn (1) & (2) it is clear that $\vec{\tau}$ is acting $\perp^r$ to the angular momentum $\vec{L}$

we know that torque $\vec{\tau}$ causes the angular momentum to change as follows

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad --- (3)$$

this change takes place in the direction of $\vec{\tau}$

Therefore a change $d\vec{L}$ in $\vec{L}$ in time dt $d\vec{L} \perp^r L$ as change in $\vec{L}$ is along $\vec{\tau}$ & $\vec{\tau} \perp^r \vec{L}$.

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The magnitude of $\vec{L}$ remains constant but direction changes. As time passes $\vec{L}$ trace a cone around $\vec{B}$ such that angle between $\vec{L}$ & $\vec{B}$ remains constant. This is the precession of $\vec{L}$ & hence of $\vec{e}$ around $\vec{B}$.

From fig-2 $\quad \omega dt = \frac{dL}{L\sin\theta}$ (3) ω = angular velocity of precession

$$\omega = \frac{dL}{dt\, L\sin\theta} = \frac{\tau}{L\sin\theta} \quad \text{--- (4)}$$

from (2) $\tau = \mu_e \times B = \mu_e B \sin\theta$ putting in eqn (4)

$$\omega = \frac{\mu_e B\sin\theta}{L\sin\theta} = \frac{\mu_e}{L} B$$

$$\therefore \boxed{\omega = \frac{\mu_e}{L} B \quad \text{--- (5)}}$$ = Angular velocity of Larmor precession

⇒ Angular velocity of Larmor precession = product of magnitude of magnetic field & the ratio of the magnitude of magnetic moment to the magnitude of the angular momentum.

we know that $\frac{\mu_e}{L} = \frac{e}{2m}$

$$\therefore \boxed{\omega = \frac{eB}{2m} \quad (6)}$$ $\quad \omega = 2\pi f$

$$2\pi f = \frac{eB}{2m} \Rightarrow \boxed{f = \frac{eB}{4\pi m} \quad \text{--- (7)}}$$ = Larmor frequency of precession.