Dr. Sanjeet Kumar, HOD Physics, HOOG

[#] Addition of Angular Momenta: Clebsch Gordan Coefficients.

Scanned with AnyScanner

(~~[illegible]~~) as obeyed by J_1 and J_2.

Say we ~~[illegible]~~ have a complete set of eigen vectors of J_1^2 and J_{1z} of system (1) and a similar complete set for system (2).

Let the eigen values of J_1^2 be $j_1(j_1+1)$, those of J_2^2 be $j_2(j_2+1)$ and those of J^2 by $j(j+1)$

Similarly let the eigen-values of J_{1z}, J_{2z} and J_z be denoted by m_1, m_2 and m respectively.

Let the simultaneous eigen-vectors of J_1^2 and J_{1z} be $\psi_1(j_1 m_1)$; of J_2^2 and J_{2z} be $\psi_2(j_2 m_2)$ and of J^2 and J_z be $\psi(j m)$.

Then we will have;

$$J_1^2\,\psi_1(j_1 m_1) = j_1(j_1+1)\psi_1(j_1 m_1)$$

$$J_{1z}\,\psi_1(j_1 m_1) = m_1\,\psi_1(j_1 m_1)$$

$$J_2^2\,\psi_2(j_2 m_2) = j_2(j_2+1)\psi_2(j_2 m_2)$$

Teacher's Signature

& $J_{2z}\,\psi_2(j_2 m_2) = m_2\,\psi_2(j_2 m_2)$ } so the products

$$\psi(jm) = \psi_1(j_1 m_1)\,\psi_2(j_2 m_2)$$

form a complete set of $(2j_1+1)(2j_2+1)$ eigen-vectors in the combined space and are simultaneous eigen-vectors of

$$J_1^2,\ J_2^2,\ J_{1z},\ J_{2z}\ \&\ J_z$$

but not of J^2. The fact that $\psi(jm)$ is an eigen-vector of J_z belonging to eigen-value $m = m_1 + m_2$ may be viewed as follows:

$$J_z\,\psi(jm) = (J_{1z} + J_{2z})\,\psi_1(j_1 m_1)\,\psi_2(j_2 m_2)$$

$$= (m_1 + m_2)\,\psi_1(j_1 m_1)\,\psi_2(j_2 m_2)$$

$$= (m_1 + m_2)\,\psi(jm)$$

Further of the three operators J_{1z}, J_{2z} and J_z, one is superfluous, hence a complete commuting set consists of J_1^2, J_2^2, J_{1z} and J_{2z}.

For combined systems J^2 and J_z are of more importance than the components. While J_1^2 and J_2^2 commute with both J^2 and J_z, but neither J_{1z} nor J_{2z} commutes with J^2. Next to find the simultaneous eigen-vectors of the set J_1^2, J_2^2, J^2 and J_z. This can be realised by changing the representation in which J_{1z} and J_{2z} are diagonal into one in which J^2 and J_z are diagonal through unitary transformation.

If $\phi(jm)$ are new eigen-vectors then they may be expressed as a linear combination of the direct product set,

i.e., $$\phi(jm) = \sum_{m_1, m_2} \langle m_1, m_2 | j, m \rangle \psi_1(j_1, m_1) \psi_2(j_2, m_2) \quad (9)$$

Where the coefficients $\langle m_1, m_2 | j, m \rangle$ are the elements of the unitary matrix affecting the transformation from a representation in which J_{1z} and J_{2z} are diagonal to one in which J^2 and J_z are diagonal and are known as Clebsch-Gordan coefficients, or vector-coupling coefficients or sometimes Wigner coefficients. These coefficients may also be represented as

$$\langle j_1 m_1 \, j_2 m_2 | j_1 j_2 j m \rangle$$

or $$\phi \left\langle \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{pmatrix} \right\rangle$$