

Paper III

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Abelian group or Commutative group: - A group  $(G, \circ)$  is said to be an abelian group if it satisfies.

Commutative law

i.e. for all  $a, b \in G$  such that  $a \circ b = b \circ a$ .

Non Abelian Group: A group  $(G, \circ)$  is said to be non abelian group if it not satisfy Commutative law.

i.e. for all  $a, b \in G$  such that

$$a \circ b \neq b \circ a.$$

Finite group: - A group  $(G, \circ)$  is called a finite group if the number of elements in the group is finite.

Infinite group: - If the group  $G$  contains an infinite number of elements. Then  $G$  is called infinite group.

Q: - Show that every group of two elements is necessarily abelian.

Solution: - A group with two elements consists of identity element  $e$  and another element  $a \neq e$ .

Since  $e^{-1} = e$  and all the elements must have inverse elements in the group  $G$ ,  $a^{-1} = a$

$$\text{Thus } G = (e, a) \text{ where } a^{-1} = a.$$

Then  $e \circ a = a \circ e = a$

i.e. Commutative law holds.

Hence  $G$  is abelian.