

II Addition of Coefficients.

System in which the total Angular Momentum is composed of two or more parts, for some applications independent of each other e.g. the particles with spin (in the non-relativistic limit), systems containing two or more particles such as many electron atoms; the scattering and radiation processes etc.

The correlation between the total angular momentum and its component parts:

Let there be two non-interfering systems ① & ② with angular momenta J_1 and J_2 , and their state vectors ψ_1 and ψ_2 . If then represented the sum of J_1 and J_2 then

$$J = J_1 + J_2 \quad \text{--- (1)}$$

If the systems ① and ② are considered together as a single system, the state vector of combination is

$$\psi = \psi_1 \psi_2 \quad \text{--- (2)}$$

Assuming the system of units is added $\hbar = 1$ or defining angular momentum Quantum Mechanically is

$$J = \hbar^{-1} (r \times p) \quad \text{--- (3)}$$

The reversal of relation of J_1 and J_2 are $J_1, \psi_1, = \hbar^{-1} J_1, \psi_1$

From these we obtain

$$\vec{J} \times \vec{J} = (\vec{J}_1 + \vec{J}_2) \times (\vec{J}_1 + \vec{J}_2)$$

$$= \vec{J}_1 \times \vec{J}_1 + \vec{J}_1 \times \vec{J}_2 + \vec{J}_2 \times \vec{J}_1 + \vec{J}_2 \times \vec{J}_2$$

$$= \vec{J}_1 \times \vec{J}_2 + \vec{J}_2 \times \vec{J}_1 \quad \left[\begin{array}{l} \text{Since } \vec{J}_1 \text{ commutes} \\ \text{with } \vec{J}_2 \end{array} \right]$$

$$= i\vec{J}_2 + i\vec{J}_2 = 2i(\vec{J}_1 + \vec{J}_2) = 2i\vec{J} \quad (5)$$

Thus \vec{J} follows the same commutation relation (eqn 4) as obeyed by \vec{J}_1 and \vec{J}_2 .