17-04-24 Dr. Sanjeet Kumar CC-3 PG Sem II

Rigid Rotator In A Fixed Plane.

Say the rotator be only confined in XY-plane, then $\theta = 90°$ (or $\pi/2$) and the S. Eqn becomes

$$\frac{\partial^2 \psi}{\partial \phi^2} + \frac{2IE}{\hbar^2}\psi = 0 \quad (1)$$

Here we have $\psi = \Phi_m(\phi)$; so that

$$\frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial \phi^2} = -\frac{2IE}{\hbar^2} = \text{Constant} = -m^2 \text{ (say)}$$

Then we will have;

$$\frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial \phi^2} = -\frac{2IE}{\hbar^2} = -m^2$$

Or $$\frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial \phi^2} + m^2 = 0 \quad (2)$$

$$\Rightarrow \frac{\partial^2 \Phi}{\partial \phi^2} + m^2 \Phi = 0$$

where

$$\frac{2IE}{\hbar^2} = m^2 \quad (3)$$

Eigen-fns: The soln. of eqn ② is given by

$$\Phi_m = A e^{im\phi} \quad (4)$$

A is any arbitrary const. & $m = 0, \pm 1, \pm 2, \ldots$

From the normalisation condition;

$$\int_0^{2\pi} \Phi_m \Phi_m^* \, d\phi = 1$$

or $$\int_0^{2\pi} A e^{+im\phi} A e^{-im\phi} \, d\phi = 1$$

or $$A^2 \cdot 2\pi = 1$$

or $$A = \frac{1}{\sqrt{2\pi}}$$

Then the eigen-fns. are given by

$$\psi = \Phi_m(\phi) = A e^{im\phi}$$

or $$\psi = \frac{1}{\sqrt{2\pi}} e^{im\phi} \quad (5)$$

Eigen-Values: From ③ we will have

$$E = E_m = \frac{m^2 \hbar^2}{2I} \quad (6)$$

→ This is the required eigen-value equation of the rigid rotator.