Binary operation:-

Let A be a non-empty set. Any function from $A \times A$ to A i.e, combines two elements of a set to produce another element of the same set is called binary operation. It is denoted by $o: A \times A \rightarrow A$ or $f: A \times A \rightarrow A$

If $f: A \times A \rightarrow A$ be a binary operation in A and $x, y \in A$ then $f(x, y)$ is composite of x and y under the composition f.

Laws of binary operations with examples:

(i) Closure law:- Let a binary operation o defined on the non-empty set A then A is closed if $a o b \in A$ for all $a, b \in A$.
e.g. The set of natural number (N) is closed under the binary operation '+'.

(ii) Associative law:- Let a binary operation o defined on the non-empty set A. Then A is associative if

$$a o (b o c) = (a o b) o c \text{ for all } a, b, c \in A.$$

i.e. Set of integers including zero is associative under ordinary operation addition '+'.

(iii) Let a binary operation 'o' defined on the non-empty set A. Then A is commutative if $a o b = b o a$ for all $a, b \in A$ ~~e.g. Set of~~
i.e. set of positive integers ~~is~~ commutative with respect to addition '+'.

(iv) Existence of identity element:- Let o be a binary operation defined on 'a'. Then for a particular element $e \in A$ such that $a o e = e o a$ for all $a \in A$. We say that the identity element exists in A for the binary operation 'o'.
i.e. Let R be the set of real number and $\times$ is the binary operation called multiplication - 1 is the identity element which exists such that $a \times 1 = 1 \times a = a$ for all $a \in R$.

2. (v) Existence of inverse element:- Let o be a binary operation defined on 'a'. There exists a corresponding element $a' \in A$ such that $a o a' = a' o a = e$. Where e is the identity element of A under the same operation 'o'. Then a is called inverse of a or a is the inverse of a'.

i.e. Let Z be the set of integers and '+' is binary operation called addition then to each $a \in Z$, we have an integer $-a \in Z$ such that $a + (-a) = (-a) + a = 0$.

(vi) Distributive law:- Let o and * are two binary operations defined over a set A such that

$$a o (b * c) = (a o b) * (a o c) \text{ for all } a, b, c \in A.$$

Then the binary operation * is distributive over the operation..

i.e. Let Z be the set of positive integer defined on Z. Then

$$a \times (b + c) = \cancel{(a \times b)}\ a \times b + a \times c$$

Where $a, b, c \in Z$.