

Mathematics
(Subsidiary) B.Sc Part II.

Successive Differentiation

To find the
 n th differential coefficient of $\sin(ax+b)$:

$$\begin{aligned} \text{Let } y &= \sin(ax+b) \\ \text{Then } y_1 &= a \cos(ax+b) \\ &= a \sin\left(ax+b+\frac{\pi}{2}\right) \cdot \underbrace{\frac{d}{dx} \sin\left(\frac{\pi}{2}+0\right)}_{= \cos 0} \end{aligned}$$

$$\begin{aligned} y_2 &= a^2 \cos\left(ax+b+\frac{\pi}{2}\right) \\ &= a^2 \sin\left(ax+b+\frac{\pi}{2}+\frac{\pi}{2}\right) \\ &= a^2 \sin\left(ax+b+2 \cdot \frac{\pi}{2}\right) \end{aligned}$$

$$\begin{aligned} y_3 &= a^3 \cos\left(ax+b+2 \cdot \frac{\pi}{2}\right) \\ &= a^3 \sin\left(ax+b+2 \cdot \frac{\pi}{2}+\frac{\pi}{2}\right) \\ &= a^3 \sin\left(ax+b+3 \cdot \frac{\pi}{2}\right) \end{aligned}$$

Let us suppose that

$$y_n = a^n \sin\left(ax+b+n \cdot \frac{\pi}{2}\right) \quad \text{--- (1)}$$

Differentiating (1), again with respect to x ,
we get

$$\begin{aligned} y_{n+1} &= a^n \cos\left(ax+b+n \cdot \frac{\pi}{2}\right) \times a \\ &= a^{n+1} \sin\left(ax+b+n \cdot \frac{\pi}{2}+\frac{\pi}{2}\right) \\ &= a^{n+1} \sin\left(ax+b+(n+1) \cdot \frac{\pi}{2}\right) \end{aligned}$$

Which is of the same form as (1) with n replaced by $(n+1)$.

Since (1) is true for $n=1, 2, 3, \dots$, therefore by the method of induction, it is true for all n .

Thus if $y = \sin(ax+b)$, then $y_n = a^n \sin\left(ax+b+\frac{n\pi}{2}\right)$

Particular case 1. If $b=0$, then $y = \sin ax$

$$\therefore y_n = a^n \sin\left(ax + \frac{n\pi}{2}\right)$$

2. If $a=1, b=0$ then $y = \sin x$.

$$\therefore y_n = \sin\left(x + \frac{n\pi}{2}\right).$$

Example - Find the n th differential coefficient of $e^x \sin^2 x$.

Solution: - Here $y = e^x \sin^2 x$

$$= e^x \frac{1 - \cos 2x}{2}$$

$$= \frac{1}{2} e^x - \frac{1}{2} e^x \cos 2x$$

\therefore From the formula

$$y_n = \frac{1}{2} e^x - \frac{1}{2} e^x (1^2 + 2^2)^{n/2} \cos\left(2x + n\left(\frac{\pi}{2}\right)\right)$$

$$= \frac{1}{2} e^x - \frac{1}{2} e^x (5)^{n/2} \cos\left(2x + n\left(\frac{\pi}{2}\right)\right)$$

$$= \frac{1}{2} e^x - \frac{1}{2} e^x (5)^{n/2} \cos\left(2x + n\left(\frac{\pi}{2}\right)\right)$$