

Mathematics (Subsidiary)
Part I.

(Equivalence Relation)

Equivalence relation: — A relation R defined on a set A is an equivalence relation iff it satisfies all the following three conditions.

(i) R is reflexive
ie. $aRa \forall a \in A$

(ii) R is symmetric
ie. $aRb \Rightarrow bRa \forall a, b \in A$

(iii) R is transitive
ie. aRb and $bRc \Rightarrow aRc \forall a, b, c \in A$.

Theorem: — Show that inverse of ~~the~~ an equivalence relation is also an equivalence relation.

Proof: — Given that R is an equivalence relation then in set X then R must be reflexive, symmetric and transitive
Let $a, b, c \in X$ then we have

$\forall a \in X$

(i) Reflexive: — $(a, a) \in R^{-1}$, $\forall (a, a) \in R \forall a \in X$
 $\Rightarrow (a, a) \in R^{-1}$
 R^{-1} is reflexive.

ii) Symmetric :- $(a, b) \in R^{-1} \Rightarrow (b, a) \in R^{-1}$
for $(a, b) \in R^{-1} \Rightarrow (b, a) \in R$
 $\Rightarrow (a, b) \in R$ ($\because R$ is symmetric)
 $\Rightarrow (b, a) \in R^{-1}$

$\therefore R^{-1}$ is symmetric.

also $(a, b) \in R^{-1} \Rightarrow (a, b) \in R \therefore R^{-1} = R$ in this case.

iii) Transitive :-

$$(a, b), (b, c) \in R^{-1}$$

$$\Rightarrow (a, c) \in R^{-1}$$

We have

$$(a, b), (b, c) \in R^{-1}$$

$$\Rightarrow (b, a), (c, b) \in R$$

$$\Rightarrow (c, b), (b, a) \in R$$

$$\Rightarrow (c, a) \in R \Rightarrow (a, c) \in R^{-1}$$

$\therefore R^{-1}$ is transitive

$\therefore R^{-1}$ is an equivalence relation in X .

proved