5 (ii). Find the sum of n terms

W.D. $\operatorname{cosec}\theta + \operatorname{cosec}2\theta + \operatorname{cosec}2^2\theta + \operatorname{cosec}2^3\theta + \cdots$ to n terms

$$t_1 = \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1+\cos\theta-\cos\theta}{\sin\theta}$$

$$= \frac{2\cos^2\theta/2}{2\sin\theta/2\cdot\cos\theta/2} - \frac{\cos\theta}{\sin\theta}$$

$$\therefore \operatorname{cosec}\theta = \cot\theta/2 - \cot\theta$$

$\therefore \operatorname{cosec}1\theta = \cot\frac{1}{2}\theta - \cot\theta$

$\operatorname{cosec}2\theta = \cot\frac{1}{2}\times 2\theta - \cot 2\theta$

$\operatorname{cosec}2^2\theta = \cot\frac{1}{2}\times 2^2\theta - \cot 2^2\theta$

$\operatorname{cosec}2^3\theta = \cot\frac{1}{2}\times 2^3\theta - \cot 2^3\theta$

$\cdots \quad \cdots \quad \cdots$

$\operatorname{cosec}2^{n-1}\theta = \cot\frac{1}{2}\times 2^{n-1}\theta - \cot 2^{n-1}\theta$

Adding $\operatorname{cosec}\theta + \operatorname{cosec}2\theta + \cdots + \operatorname{cosec}2^{n-1}\theta = \cot\theta/2 - \cot 2^{n-1}\theta$ Ans

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin 2A = 2\sin A\cos A$, $\sin A = 2\sin A/2\cos A/2$

$\frac{6.}{\text{w.o.}}$ $\dfrac{1}{\cos\phi+\cos 3\phi}+\dfrac{1}{\cos\phi+\cos 5\phi}+\dfrac{1}{\cos\phi+\cos 7\phi}+\cdots$ to n terms

$$T_r=\frac{1}{\cos\phi+\cos(2r+1)\phi}$$

(3, 5, 7, … का r वाँ पद $=3+(r-1)2$
$=3+2r-2=(2r+1)$

or
$$T_r=\frac{1}{2\cos\frac{(2r+1)\phi+\phi}{2}\cos\frac{(2r+1)\phi-\phi}{2}}\quad\left[\cos C+\cos D=2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right.$$

$$=\frac{1}{2\cos\frac{(2r+2)}{2}\phi\cos\frac{2r\phi}{2}}$$

$$T_r=\frac{1}{2\cos(r+1)\phi\cdot\cos r\phi}$$

$$=\frac{1}{2\sin\phi}\times\frac{\sin\phi}{\cos(r+1)\phi\cdot\cos r\phi}$$

$$=\frac{1}{2\sin\phi}\times\frac{\sin\{(r+1)\phi-r\phi\}}{\cos\{(r+1)\phi\}\cdot\cos r\phi}$$

$$=\frac{1}{2\sin\phi}\times\frac{\sin(r+1)\phi\cos r\phi-\cos(r+1)\phi\sin r\phi}{\cos(r+1)\phi\cdot\cos r\phi}$$

$$=\frac{1}{2\sin\phi}\left[\frac{\sin(r+1)\phi\cos r\phi}{\cos(r+1)\phi\cdot\cos r\phi}-\frac{\cos(r+1)\phi\sin r\phi}{\cos(r+1)\phi\cdot\cos r\phi}\right]$$

$$=\frac{1}{2\sin\phi}\left[\tan(r+1)\phi-\tan r\phi\right]$$

$$\therefore T_r=\frac{1}{2\sin\phi}\left[\tan(r+1)\phi-\tan r\phi\right]$$

$$T_1=\frac{1}{2\sin\phi}\left[\tan 2\phi-\tan\phi\right]$$

$$T_2=\frac{1}{2\sin\phi}\left[\tan 3\phi-\tan 2\phi\right]$$

$$T_3=\frac{1}{2\sin\phi}\left[\tan 4\phi-\tan 3\phi\right]$$

$$\cdots\cdots\cdots$$

$$T_n=\frac{1}{2\sin\phi}\left[\tan(n+1)\phi-\tan\phi\right]$$

$\therefore S = \frac{1}{2\sin 2\phi}\left[\tan(n+1)\phi - \tan\phi\right]$ ③

EXERCISE - 7

① $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{4}{33} + \cdots + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}}$

$t_n = \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} = \tan^{-1}\frac{2^{n-1}(1)}{1+2^n\cdot 2^{n-1}}$

$t_n = \tan^{-1}\frac{2^{n-1}(2-1)}{1+2^n\cdot 2^{n-1}} = \tan^{-1}\frac{2\cdot 2^{n-1} - 2^{n-1}}{1+2^n\cdot 2^{n-1}}$

$= \tan^{-1}\frac{2^n - 2^{n-1}}{1+2^n\cdot 2^{n-1}}$

$t_n = \tan^{-1}2^n - \tan^{-1}2^{n-1}$

$t_1 = \tan^{-1}2 - \tan^{-1}2^0$

$t_2 = \tan^{-1}2^2 - \tan^{-1}2^1$

$t_3 = \tan^{-1}2^3 - \tan^{-1}2^2$

$\vdots$

$t_{n-1} = \tan^{-1}2^{n-1} - \tan^{-1}2^{n-2}$

$t_n = \tan^{-1}2^n - \tan^{-1}2^{n-1}$

Adding $S = \tan^{-1}2^n - \tan^{-1}1 = \tan^{-1}2^n - \pi$