

B.Sc Part II (Hons) Paper I

Cartesian product of Sets.

Definition: —

Ordered pair: — An element of the form (x, y) is known as ordered pair where x, y belongs to the any given set A .

The element x is called first element and y is called second element respectively.

$$\text{i.e. } (x, y) = (y, x).$$

Cartesian product of two sets: — Let X and Y be any two sets. Let (x, y) be ordered pair, then Cartesian product of x and y denoted by $X \times Y$ (read as X cross Y and defined as $X \times Y = \{(x, y) : x \in X, y \in Y\}$ i.e. the first element of ordered pair belongs to the first set and second element of ordered pair belongs to the second set.

Cartesian product in general form: — Let $A_1, A_2, A_3, \dots, A_n$ be n given sets. The set of ordered n -tuples $(a_1, a_2, a_3, \dots, a_n)$, $a_i \in A_i$ for $i = 1, 2, 3, \dots, n$ is called the Cartesian product $A_1, A_2, A_3, \dots, A_n$ and is denoted by $A_1 \times A_2 \times A_3 \times \dots \times A_n$.

Cartesian product of an indexed family of sets:-

Let $(A_i: i \in I)$ be an indexed family of non-empty set A_i . Then the Cartesian product of this family denoted by $\prod_{i \in I} A_i$ is defined to be the set of all functions $(x: \prod_{i \in I} A_i)$ defined over I and taking values in $\prod_{i \in I} A_i$ such that $x_i \in A_i$ for each $i \in I$.

Theorem 1: - Show that $A \subseteq B \Rightarrow A \times A = (A \times B) \cap (B \times A)$

Proof: - $\textcircled{1}$ Given that $A \subseteq B \Rightarrow A \cap B = A$ ——— $\textcircled{1}$

We have $A \times A = (A \cap B) \times (A \cap B)$ from $\textcircled{1}$

$$= (A \cap B) \times (B \cap A) \\ \text{(By commutative law)}$$

$$\text{But } (A \times B) \cap (C \times D) = \frac{A \cap C}{A \cap C} \times (B \cap D)$$

$$\Rightarrow (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$$

Replace C by B and D by A , we get

$$(A \cap B) \times (B \cap A) = (A \times B) \cap (B \times A)$$

$$\Rightarrow A \times A = (A \times B) \cap (B \times A) \text{ (using } \textcircled{1} \text{ for } A \cap B = A)$$