

Hypothesis Testing

M.A Semester-II

CC-9

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Statistical Hypothesis

A statistical hypothesis is some assumption or statement, which may or may not be true, about a population or equivalently about the probability distribution characterising the given population, which we want to test on the basis of the evidence from a random sample.

If hypothesis completely specifies the population, then it is known as simple hypothesis, otherwise it is known as composite hypothesis.

Null Hypothesis

According to Prof. A.R.Fisher: Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true.

It is usually denoted by H_0 .

How to set up a Null Hypothesis?

It is always taken as a hypothesis of no difference. The statistician should always adopt the neutral or null attitude regarding the outcome of sample.

The following points may be borne in mind in setting the null hypothesis:

1. (a) Express the claim or hypothesis to be tested in the symbolic form.
(b) Write the symbolic form if the original claim or hypothesis is not true.
(c) Identify the null hypothesis and the alternative hypothesis as explained below:

In steps (a) and (b) above, take the expression involving equality sign as the null hypothesis (H_0) and the other as the alternative hypothesis (H_1).

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2. If we want to test the significance of the difference between a statistic and the parameter, or between two sample statistics, then we set up the null hypothesis H_0 that the difference is not significant.
 3. If we want to test any statement about the population, we set up the null hypothesis that it is true.

Alternative Hypothesis

Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis.

It is usually denoted as H_1 .

Example:

If we want to test the null hypothesis that the population has specified mean μ_0

$$H_0: \mu = \mu_0$$

then the alternative hypothesis could be:

- (i) $H_1: \mu \neq \mu_0$ ($\mu > \mu_0$ or $\mu < \mu_0$) – *two tailed alternative hypothesis*
- (ii) $H_1: \mu > \mu_0$ – *right tailed alternative hypothesis*
- (iii) $H_1: \mu < \mu_0$ – *left tailed alternative hypothesis*

Types of Errors in Testing of Hypothesis

	Decision from Sample	
	Reject H_0	Accept H_0
H_0 True	Wrong(Type I Error)	Correct
H_0 False(H_1 True)	Correct	Wrong(Type II Error)

Level of Significance

The maximum size of the type I error, which we are prepared to risk is known as the *level of Significance*. It is usually denoted by α and is given by:

$$P[\text{Rejecting } H_0 \text{ when } H_0 \text{ is true}] = \alpha$$

Commonly used level of significance in practice are 5%(0.05) and 1%(0.01). If we adopt 5% level of significance, it implies that in 5 samples out of 100, we are likely to reject a correct H_0 . In other words, this implies that we are 95% confident that our decision to reject H_0 is correct.

Procedure for Testing Hypothesis

1. Rejection Region Method
2. P-value Testing Method

Rejection Region Method

Step 1: Set up the Null Hypothesis.

Step 2: Set up the Alternative Hypothesis.

Step 3: Choose the appropriate level of significance.

Step 4: Identify the sample statistic to be used and its sampling distribution.

Step 5: Define and compute the test statistic under H_0 .

Step 6: Obtain the critical value.

Step 7: If the computed value of the test statistic lies in the rejection region, we fail to reject H_0 .

If the computed value of test statistic lies outside the rejection region, we fail to reject H_0 .

P-Value Estimation Method

Step 1 to 5 are the same as in method 1.

Step 6: Find the P-value of the computed test statistic under H_0 in step 5.

Step 7: If $P\text{-value} < \alpha$, we reject H_0 at ' α ' level of significance.

If $P\text{-value} > \alpha$, we fail to reject at ' α ' level of significance.

Critical Region

Suppose we take several samples of the same size from a given population and compute some statistic for each of the sample. Let $t_1, t_2 \dots t_k$ be the values of the statistic for these samples. Each of these values may be used to test some null hypothesis H_0 . Some values may lead to rejection of H_0 while others may lead to acceptance of H_0 . The statistic which leads to rejection of H_0 give us a region called *Critical Region or rejection Region*, while those which lead to the acceptance of H_0 give us a region called *Acceptance Region*. It depends upon the level of significance and the alternative hypothesis, whether it is two- tailed or single-tailed.

P-Value

The probability that the value of test statistic is at least as extreme as its computed value on the basis of the sample data under H_0 , is called its *P-Value*.