

D2(1) Maths
paper-III, Shankar's Method of finding the auxiliary 1/4
Group-B. We have seen in the comparison test that

If $\lim_{n \rightarrow \infty} \frac{u_n}{v_n} = l$ where l is non-zero finite number

then the two $\sum u_n$ and $\sum v_n$ either converge or diverge. Hence we infer that the convergence or divergence of $\sum u_n$ depends upon the convergence or divergence of $\sum v_n$. i.e. if $\sum v_n$ is convergent then $\sum u_n$ will also be convergent and if $\sum v_n$ is divergent then the series $\sum u_n$ will also be divergent.

The series $\sum v_n$ is called an Auxiliary series. To find the convergence or divergence of a series whose n th term is u_n , we must know the nature of the auxiliary series $\sum v_n$.

A method of finding v_n is given below.

Let us take u_n , the n th term of the given series and retain only the highest powers of n in both numerator and denominator.

Let us denote this part by v_n and $u_n = \text{a constant} + \text{some powers of } n$.

Ex 1 consider $u_n = \frac{n^2+1}{n^3+2}$

Here, the term containing the highest power of n in the numerator $= n^2$ and in the denominator $= n^3$.

\therefore we take $v_n = \frac{n^2}{n^3} = \frac{1}{n}$

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prove that the series

$$1 + \frac{4}{5} + \frac{6}{10} + \frac{8}{15} + \dots + \frac{2n}{n^2+1} + \dots$$

Sol. Here $u_n = \frac{2n}{n^2+1} = \frac{2n}{n(1+\frac{1}{n})} = \frac{2}{1+\frac{1}{n}}$

Let us take an auxiliary series with term $v_n = \frac{1}{n}$

$$\therefore \frac{u_n}{v_n} = \frac{2}{n(1+\frac{1}{n})} \cdot n = \frac{2}{1+\frac{1}{n}}$$

$$\therefore \lim_{n \rightarrow \infty} \frac{u_n}{v_n} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1} = 2$$

By comparison test both the series $\sum u_n$ and $\sum v_n$ will be of the same nature.

But the series $\sum v_n$ whose n th term $v_n = \frac{1}{n}$ is divergent, so the given series $\sum u_n$ is also divergent.

(3) Examine the series

$$1 + \frac{3}{4} + \frac{5}{13} + \frac{7}{25} + \dots + \frac{2n-1}{2n^2-2n+1} + \dots$$

Sol. Here, we have

$$u_n = \frac{2n-1}{2n^2-2n+1} = \frac{1}{n} \frac{(2-\frac{1}{n})}{(2-\frac{2}{n}+\frac{1}{n^2})}$$

(3) Let us consider an arbitrary $\epsilon > 0$
whose nth term v_n is

$$\therefore \frac{u_n}{v_n} = \frac{2 - \frac{1}{n}}{2 - \frac{1}{n} + \frac{1}{n^2}}$$

Now take a limit when $n \rightarrow \infty$, we have

$$\lim_{n \rightarrow \infty} \frac{u_n}{v_n} = \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{2 - \frac{1}{n} + \frac{1}{n^2}} = \frac{2}{2} = 1 \quad \text{finite and non-zero}$$

By comparison test, both the series $\sum u_n$ and $\sum v_n$ will be of the same nature. Here $\sum v_n$ is divergent, so $\sum u_n$ is also divergent.