D.C.(H) (Maths) Paper III 15/4/2021

Problem based on Lebnitz' Test ①

Example:- Discuss the convergence and divergence of the following infinite series

(a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

This is an alternating series

Without considering sign,

$u_n = \frac{1}{n}, \quad u_{n+1} = \frac{1}{n+1}$

$\therefore u_n > u_{n+1}$

Terms are decreasing.

$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n} = 0$

Hence according to Leibnitz Test

$\sum u_n$ is convergent.

2. Test the convergency of the series

$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$

This is an alternating series

Without considering sign

$u_n = \frac{1}{\sqrt{n}}, \quad u_{n+1} = \frac{1}{\sqrt{n+1}}$

clearly $u_n > u_{n+1}$

$\therefore$ terms are decreasing.

$\lim_{n\to\infty} u_n = \frac{1}{\sqrt{n}} \to 0$

Scanned with Cam

Hence according to Leibnitz test
$\sum u_n$ is convergent

(2)

(2) Test the convergency of the series $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\cdots$

Here $u_n = \frac{n+1}{n}$

and $u_{n+1} = \frac{n+2}{n+1}$

~~Clearly~~

$$u_n - u_{n+1} = \frac{n+1}{n} - \frac{n+2}{n+1}$$

$$= \frac{(n+1)^2 - n(n+2)}{n(n+1)}$$

$$= \frac{\cancel{n^2} + \cancel{2n} + 1 - \cancel{n^2} - \cancel{2n}}{n(n+1)}$$

$$= \frac{1}{n(n+1)} > 0$$

$\therefore u_n > u_{n+1}$

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty}\left(\frac{n}{n} + \frac{1}{n}\right) = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$

$$= 1+0 = 1 \neq 0$$

$\lim_{n\to\infty} u_n = 0$ is not satisfied. and

Hence the series is not convergent.