Expression for Average velocity :→

The average velocity or mean velocity of molecules obeying Maxwell laws defined by

$$C_{av} = \frac{1}{n}\int_0^\infty c \, dn_c$$

where $dn_c = 4\pi n a^3 e^{-bc^2} c^2 dc$.

or. $C_{av} = \frac{1}{n} 4\pi n a^3 \int_0^\infty e^{-bc^2} c^3 dc$

$\int_0^\infty e^{-bc^2} c^3 dc = ? = \frac{1}{2b^2}$

$= 4\pi a^3 \left[\frac{1}{2b^2}\right]$

(Where the value of integral is square bracket)

$= 4\pi a^3 \times \frac{1}{2b^2} = 2\pi \frac{a^3}{b^2}$

C_{av} or $\bar{C}$

Now $a = \sqrt{b/\pi}$

Then $C_{av} = 2\pi \frac{b^{3/2}}{\pi^{3/2}} \times \frac{1}{b^2}$

$= 2\sqrt{\frac{1}{\pi b}} = 2\sqrt{\frac{2kT}{m\pi}}$

$= \sqrt{\frac{8kT}{m\pi}}$

$= \sqrt{\frac{2.55}{m}kT} = 1.596\sqrt{\frac{kT}{m}}$

$a^3 = \frac{b^{3/2}}{\pi^{3/2}}$

and $b = \frac{m}{2kT}$

where m = molecular mass
T = absolute temp
k = Boltzmann Const.

Root Mean Square velocity :—

The root mean square velocity is the root of average of square of velocity and is defined by

$$C^2_{r.m.s} = \frac{1}{n}\int_0^\infty c^2 dn_c$$

$$= \frac{1}{\cancel{n}} 4\pi \cancel{n} a^3 \int_0^\infty e^{-bc^2} c^4 dc$$

$$= 4\pi a^3 \times \left[\sqrt{\frac{9\pi}{64 b^5}}\right]$$

$$= \cancel{4}\pi a^3 \times \frac{3}{\cancel{8}_2}\sqrt{\pi/b^5}$$

$$= \frac{3}{2}\pi a^3 \sqrt{\pi/b^5}$$

$$= \frac{3}{2}\times\frac{1}{b} = \frac{3}{\cancel{2}}\cdot\frac{\cancel{2}kT}{m} = \frac{3kT}{m}$$

$\int_0^\infty e^{-bc^2} c^4 dc = ?$

$\therefore \frac{C_{av}}{C_{rms}} = \frac{1.596\sqrt{kT/m}}{1.732\sqrt{kT/m}}$

$= \frac{1.596}{1.732} = 0.921$

or $C_{av} = 0.921\, C_{rms}$

Therefore $C_{r.m.s} = \sqrt{\frac{3kT}{m}} = 1.732\sqrt{\frac{kT}{m}}$

Most probable velocity :—

The most probable velocity is that which most of the molecules have. Therefore it is obtained by differentiating $f(c)$ with respect to c and equating the result to zero.

or which have max no. of molecules

Hence $\frac{d}{dc} f(c) = 0$

where $f(c) = 4\pi n a^3 e^{-bc^2} c^2 dc$

$$= A\, e^{-bc^2} c^2\, dc$$

where $A = 4\pi n a^3$

Hence $\frac{d}{dc} f(c) = A\left[2c\, e^{-bc^2} + c^2 e^{-bc^2} \times -2bc\right]$

$$= 0$$

or, $2c\, e^{-bc^2} \times \{1 - bc^2\} = 0$

but $c \neq 0$

Hence $bc^2 = 1$

or $c^2 = 1/b$

or $c = \frac{1}{\sqrt{b}}$

Therefore $C_{mp} = \sqrt{\frac{2kT}{m}}$

Most probable velocity $= \sqrt{\frac{2kT}{m}}$

$= 1.414\sqrt{\frac{kT}{m}}$

$\frac{C_{mp}}{C_{rms}} = \frac{1.414\sqrt{kT/m}}{1.732\sqrt{kT/m}}$

$= \frac{1.414}{1.732} = 0.816$

$C_{mp} = 0.816\, C_{rms}$

Thus we see that

$$C_{r.m.s} > C_{av} > C_{mp}.$$

The end