Ques:- Deduce maxwell's distribution law of velocities for a gas molecules.

Ans:-

According to Kinetic Theory gases consists of molecules which are endo

र्षित → -wed with Thermal motion and are in chaotic

यवस्थित हालत condition. The molecules possess all velocities from 0 to ∞. The distribution law derived by Maxwell (and developed by Boltzmann) aims at finding the number of molecules in a unit volume which lie between velocity range c and $c+dc$, where c is r.m.s. velocity.

For this maxwell made these two assumptions :-

(I) The no. of molecules per unit volume in any region under Steady Condition is Constant.

(II) In every elementary volume molecules have velocities in all directions and their magnitude is distributed according to the certain law, which we propose to derive. As the number of molecules, even in a small volume is large. The problem is thus statistical.

Maxwell produced on the above two assumptions and applied the idea of probabilities to derive the Statistical law.

Let us consider an elementary volume du, dV and dw of a gas in

Cartisian co-ordinates where the molecular density is n and velocity component $P(x,y,z)$ are u, v, w respectively.

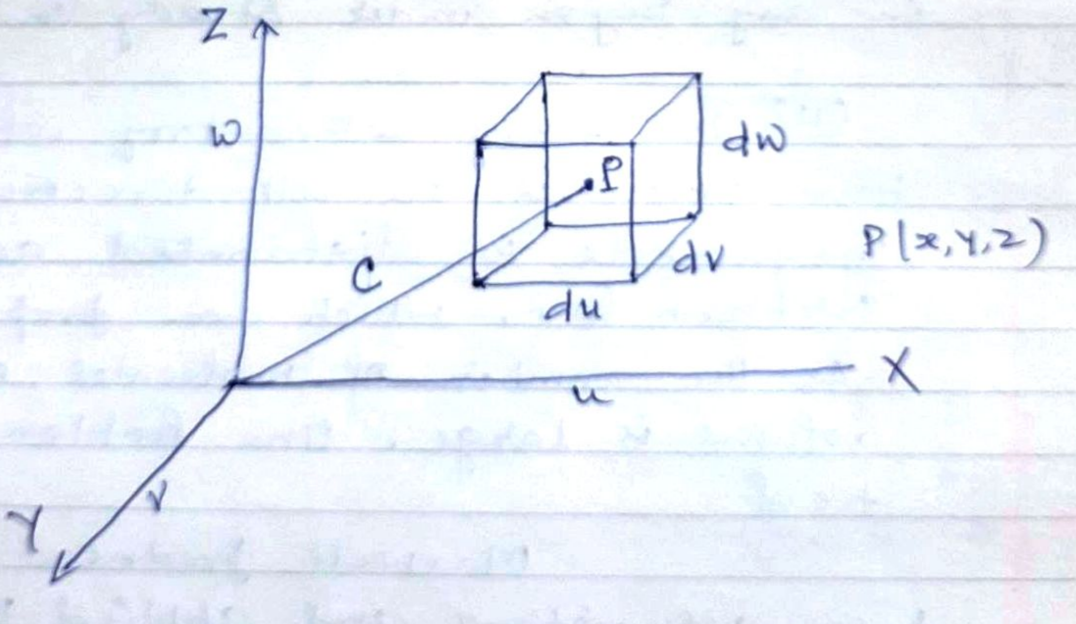
We have to find the number of molecules lying between u & $u+du$, v and $v+dv$, w and $w+dw$.

If u, v, w be the components of c at point P, then we have to find the number of molecules between c & $c+dc$.

First considering x- component :-

We see that the number of molecules lying between u & $u+du$ is some function of u (say), $nf(u)$.

Where n is molecular density and $f(u)$ is a function to be determined.



Selecting one molecule moving at random in the elementary volume. We have the probability for it to lie between velocity range u and $u+du$ as a function of u say $f(u)\,du$. Since the velocity components are mutually perpendicular & they are independent of each other. Hence the probability for the molecules to have velocity lying between v and $v+dv$ is $f(v)\,dv$ and that between w & $w+dw$ is $f(w)\,dw$.

Hence the probability that the velocity components of the selected molecule will lie between u & $u+du$, v & $v+dv$ and w & $w+dw$. Simultaneously given by the product.

ie the composite probability $= f(u)\,f(v)\,f(w)\,du\,dv\,dw$

Hence the no. of such molecules
$$= n\,f(u)\,f(v)\,f(w)\,du\,dv\,dw$$

and these are contained in volume $du\,dv\,dw$.

These molecules have a resultant velocity c where
$$c^2 = u^2+v^2+w^2 \quad \text{—— (A)}$$

Hence their no. is also $n f(c)\,du\,dv\,dw$

Where $f(c)$ is a function of c given by the relation of u, v, w.

Thus
$$n f(u)\,f(v)\,f(w) = n F(c) = n\,\phi\,(u^2+v^2+w^2)$$
$$= n\,\phi\,c^2 \quad \text{—— (1)}$$

where ϕ is another function.

To solve it we have
$C =$ constant at const. temp

$\therefore \quad d(C^2) = 0$ from eqn (A)

or, $d(u^2+v^2+w^2) = 0$

or, $u\,du + v\,dv + w\,dw = 0$ — (2)

Now taking differential of eqn (1) both sides, we have

$$f'(u)\,du\,f(v)\,f(w) + f'(v)\,dv\,f(u)\,f(w) + f'(w)\,dw\,f(u)\,f(v) = 0$$

where dashes represents 1st derivative

Dividing above eqn by $f(u)f(v)f(w)$
we have

$$\frac{f'(u)}{f(u)}du + \frac{f'(v)}{f(v)}dv + \frac{f'(w)}{f(w)}dw = 0 \quad \text{— (3)}$$

Multiplying eqn (2) by constant β both sides we get—

$$\beta u\,du + \beta v\,dv + \beta w\,dw = 0 \quad (3i)$$

Now combining (3) & (3i) by Laplaces method of undetermined multipliers
we have.

$$\left(\frac{f'(u)}{f(u)} + \beta u\right)du + \left(\frac{f'(v)}{f(v)} + \beta v\right)dv + \left(\frac{f'(w)}{f(w)} + \beta w\right)dw = 0 \quad \text{— (4)}$$

identity (स्वाभाविकता)

Now this expression is an identity. Hence separate terms must vanish.

Taking 1st term we get

$$\left(\frac{f'(u)}{f(u)} + \beta u\right) du = 0$$

or, $$\frac{f'(u)}{f(u)} du = -\beta u \, du$$

Integrating we get

$$\log f(u) = -\beta \frac{u^2}{2} + \text{Const.}$$

where $\log_e e = 1$, $a = \text{const.}$, $\log_e 1 = 0$, $\log m^n = n \log m$

$$= -\beta \frac{u^2}{2} \log_e e + \log a$$

$$= \log e^{-\frac{\beta u^2}{2}} + \log a$$

$$= \log_e e^{-bu^2} + \log a$$

where $b = \beta/2$

$$\text{or } \log f(u) = \log a e^{-bu^2}$$

~~Similarly~~ or $f(u) = a e^{-bu^2}$

Similarly $f(v) = a e^{-bv^2}$

and $f(w) = a e^{-bw^2}$

These function are determined and we have

$$f(u) f(v) f(w) = a^3 e^{-b(u^2+v^2+w^2)}$$

And the number of molecules lying between velocity range u and $u+du$, v and $v+dv$, w & $w+dw$ in elementary volume is given by

$$dn = n\, a^3\, e^{-b(u^2+v^2+w^2)}\, du\, dv\, dw$$

The Constant $a = \sqrt{b/\pi}$ and $b = \frac{m}{2KT}$

where K = Boltzmann Constant
T = absolute temp.
m = mass of a molecule

Hence

$$dn = n \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\frac{m}{2}\left(\frac{u^2+v^2+w^2}{KT}\right)} du\, dv\, dw \quad \text{—— (5)}$$

We can now obtain the number of molecules lying between c & $c+dc$. Where we have the volume $du\, dv\, dw$ is velocity space and in place of this elementary volume. The volume element in spherical polar co-ordinates (r, θ, ϕ)

$$= dc \cdot c\, d\theta \cdot c \sin\theta\, d\phi$$

and may be substituting for $du\, dv\, dw$ in eqn (5) and velocities are now between c & $c+dc$, θ & $\theta + d\theta$, ϕ & $\phi + d\phi$.

To find the total no. of molecules between c & $c+dc$, we have to integrate the expression for all possible value of ϕ & θ.

Hence

$$dn_c = n\, a^3\, e^{-bc^2}\, c^2\, dc \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \cdot d\theta$$

$= [\phi]_0^{2\pi}$
$2\pi - 0 = 2\pi$
$\ldots d\theta = [-\cos\theta]_0^{\pi}$
$[\cos\pi - \cos 0]$
$[-1 - 1]$
$[-2]$
$= +2$

$$dn_c = 4\pi n a^3 e^{-bc^2} c^2\, dc \quad \text{—— (6)}$$

which is the distribution law of Maxwell.

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Now putting $bc^2 = x^2$

we have

$$dn_c = 4n\pi^{-1/2} x^2 e^{-x^2} dx$$

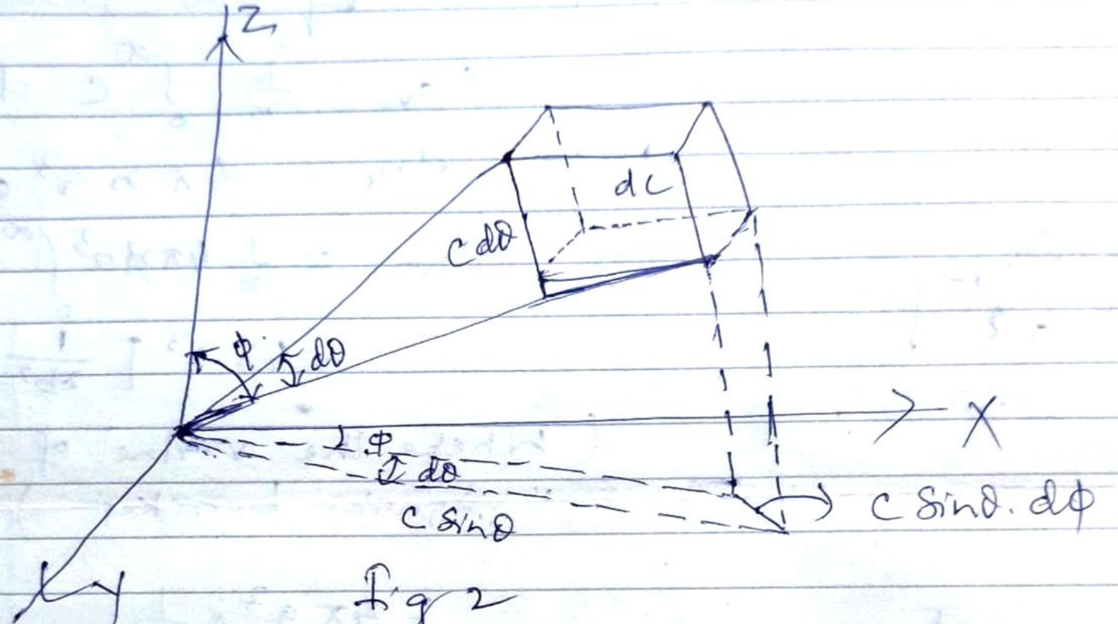
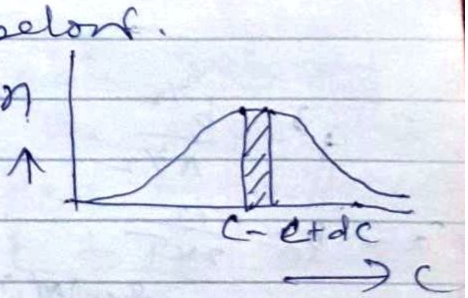


fig 2

If we plot the ~~function~~ for $y = 4\pi^{-1/2} e^{-x^2} x^2$ ~~graph~~ against x, we get the Maxwellian curves as shown in fig below.



And the number of molecules dn_c whose speed lies between u & $u+du$ is proportional to shadowed area.

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